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lit: ~~Demlová, Pondělíčková: Úvod do algebry~~

math.feld.cvut.cz → seznam předmětů → ÚA

pdf lineární algebra - pro hlubší studium

zápočet - aktivní účast, 3 omluvené absence (stačí uštné)

- 3 kontrolní testy 6., 11. a 13. týden

po 2 příkladech - max. 60 bodů - dobrovolně

- písemka - 6 příkladů - max 60 bodů

→ je jednoho alespoň 30 b, bere se vyšší počet

- ústní zk. 0-20 bodů

→ max 80 b.

1. 65-80 b

2. 50-64 b

3. 30-49 b

<http://pitbul.wz.cz>

polynom

$$a_0, a_1, a_2, \dots, a_n : \forall x \in \mathbb{R} \quad P(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

$$(2 - 3x + 5x^2) \cdot (3x - x^2 + 2x^3) = \underline{6x} - \underline{9x^2} + \underline{15x^3} - \underline{2x^2} + \underline{3x^3} - \underline{5x^4} + \\ + \underline{4x^3} - \underline{6x^4} + \underline{10x^5} = 6x - 11x^2 + 22x^3 - 11x^4 + 10x^5$$

stupen polynomu

$$\deg(P \pm Q) \leq \max(\deg P, \deg Q)$$

$$\deg(c \cdot P) = \deg P$$

$$\deg(P \cdot Q) = \deg P + \deg Q$$

dělení

$$P(x) = x^6 + 2x^5 + x^4 + 2x^3 + x$$

$$Q(x) = x^2 + 1$$

$$(x^6 + 2x^5 + x^4 + 2x^3 + x) : (x^2 + 1) = \underbrace{x^4 + 2x^3 + \frac{x}{x^2+1}}_{Y(x)}$$

$$- (x^6 + x^4)$$

$$2x^5 + 2x^3 + x$$

$$- (2x^5 + 2x^3)$$

x

$$Z(x) = x$$

$$P(x) = Q(x) \cdot Y(x) + Z(x)$$

$$(x^4 - x^3 + x^2 - 2x) : (x - 1) = x^3 + x \text{ ~~1~~ } - 1 - \frac{1}{x-1}$$

$$- (x^4 - x^3)$$

$$x^2 - 2x$$

$$- (x^2 - x)$$

$$-x$$

$$- (-x + 1)$$

$$-1$$

$$P(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$P(x) = (((a_4 x + a_3)x + a_2)x + a_1)x + a_0$$

Hornerovo schema

$$\begin{array}{rcccccc} a_4 & a_3 & a_2 & a_1 & a_0 & \\ & b_4 \cdot c & b_3 \cdot c & b_2 \cdot c & b_1 \cdot c & \\ \hline b_4 & b_3 & b_2 & b_1 & \boxed{b_0} & = P(c) \end{array}$$

$$b_4 = a_4$$

$$b_3 = a_3 + b_4 \cdot c$$

$$b_0 = a_0 + b_1 \cdot c$$

→ snadné vyčíslení hodnoty polynomu v bodě c

$$P(x) = 2x^4 - 3x^3 + 5x^2 - x + 6 \quad \text{v bodě } -2$$

$$\begin{array}{rcccccc} 2 & -3 & 5 & -1 & 6 & \\ & -4 & 14 & -38 & 78 & \\ \hline 2 & -7 & 19 & -39 & 84 & \end{array}$$

$$P(x) = x^4 - x^3 + x^2 - 2x, \quad Q(x) = x - 1$$

u1.

$$\begin{array}{rcccccc} 1 & -1 & 1 & -2 & 0 & \\ & 1 & 0 & 1 & -1 & \\ \hline 1 & 0 & 1 & -1 & -1 & \\ x^3 & +x & -1 & & & \end{array}$$

→ kořen 1, použij do Hornerova schématu

↓
rychle vyčísleno
(příklad ze strany 2)

kóřen polynomu - po jeho dosazení vyjde 0
každý polynom alespoň 1. stupně má alespoň 1 komplexní kóřen

~~polynom~~

$$P(x) = x^5 + 8x^4 + 7x^3 - 52x^2 - 44x + 80$$

$$P(0) = 80$$

nejdříve metoda pokus/omyšl

$$P(-1) = 72$$

$$\begin{array}{r} 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ -1 \quad -7 \\ \hline 1 \quad 7 \quad 0 \quad \dots \quad 72 \end{array}$$

na tabuli dělá
rychleji

$$P(1) = 0 ; \underline{x_1 = 1}$$

$$\begin{array}{r} P(2) = 0 \quad 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ \quad \quad 2 \quad 20 \quad 34 \quad 4 \quad -80 \\ \hline 1 \quad 10 \quad 27 \quad 2 \quad -40 \quad 0 \end{array}$$

$$\begin{array}{r} P(-2) = 0 \quad 1 \quad 8 \quad 7 \quad -52 \quad -44 \quad 80 \\ \quad \quad -2 \quad -12 \quad 10 \quad 84 \quad -80 \\ \hline 1 \quad 6 \quad -5 \quad -42 \quad 40 \quad 0 \end{array}$$

lim. kórkami
nějaké kórkony

$$P(x) = (x-1)(x-2)(x+2) \cdot Q(x)$$

$$(x-1)(x^2-4) = x^3 - x^2 - 4x + 4$$

jak tím dělíme do původní
a kórkáme sbylé

$$(x^5 + 8x^4 + 7x^3 - 52x^2 - 44x + 80) : (x^3 - x^2 - 4x + 4) = x^2 + 9x + 20 = (x+4)(x+5)$$

$$\begin{array}{r} (x^5 + 8x^4 + 7x^3 - 52x^2 - 44x + 80) \\ - (x^3 - x^2 - 4x + 4) \\ \hline 9x^4 + 11x^3 - 56x^2 - 44x + 80 \\ - (9x^4 - 9x^3 - 36x^2 + 36x + 36) \\ \hline 20x^3 - 20x^2 - 80x + 44 \\ - (20x^3 - 20x^2 - 80x + 80) \\ \hline 0 \end{array}$$

$$P(x) = (x-1)(x-2)(x+2)(x+4)(x+5)$$



najít kořeny rovnice

$$8x^4 - 12x^3 + 9x^2 + 12x - 17 = 0$$

$$x_1 = \frac{3}{4} + \frac{5}{4}i \text{ ten nám uveď}$$

$$x_2 = \frac{3}{4} - \frac{5}{4}i \text{ to musí být}$$

$$(x - \frac{3}{4} - \frac{5}{4}i)(x - \frac{3}{4} + \frac{5}{4}i) =$$

$$= x^2 + (-\frac{3}{4} + \frac{5}{4}i)x + (-\frac{3}{4} - \frac{5}{4}i)x +$$

$$+ (-\frac{3}{4} - \frac{5}{4}i) \cdot (-\frac{3}{4} + \frac{5}{4}i) =$$

$$= x^2 - \frac{6}{4}x + \frac{34}{16} = x^2 - \frac{3}{2}x + \frac{17}{8}$$

a dělíme

$$(8x^4 - 12x^3 + 9x^2 + 12x - 17) : (x^2 - \frac{3}{2}x + \frac{17}{8}) = 8x^2 - 8x$$

$$- (8x^4 - 6x^3 + \frac{17}{2}x^2)$$

$$= -6x^3 + \frac{1}{2}x^2 + 12x - 17$$

$$- (-6x^3 + 9x^2 - \frac{57}{4}x)$$

$$= -\frac{17}{2}x^2 + \frac{99}{4}x$$

$$= P(x^2 - 1)$$

$$x_3 = 1$$

$$x_4 = -1$$

$$P(x) = (x^5 - 4x^3 - 2x^2 + 3x + 2)$$

ve tvaru součinu kořenů
činitelů

$$P(1) = 0$$

$$P(-1) = 0$$

$$P(2) = 0$$

$$(x-1)(x+1)(x-2) = (x^2-1)(x-2) = x^3 - 2x^2 - x + 2$$

$$(x^5 - 4x^3 - 2x^2 + 3x + 2) : (x^3 - 2x^2 - x + 2) = x^2 + 2x + 1$$

$$- (x^5 - 2x^4 - x^3 + 2x^2)$$

$$= 2x^4 - 3x^3 - 4x^2 + 3x + 2$$

$$- (2x^4 - 4x^3 - 2x^2 + 4x)$$

$$= 6x^3 - 2x^2 - x + 2$$

$$= (x+1)^2$$

$$P(x) = (x+1)^3(x-1)(x-2)$$

$$3\text{-násobný kořen } -1$$

DÚ : vyřešit

1) $3x^3 - 7x^2 - 7x + 3 = 0$

2) $6x^5 - 11x^4 - 33x^3 + 33x^2 + 11x - 6 = 0$

Racionální lomená fce
podíl 2 polynomů

$$\frac{P(x)}{Q(x)} \quad ; \quad Q(x) \neq 0$$

rychlí lomená fce : $\text{stp } P < \text{stp } Q$

rozklad:

a) $\frac{a}{(x-\alpha)^n} : a \in \mathbb{R}$ reál. koef. o násobnosti n
b) $\frac{Bx+C}{(x^2+px+q)^m} ; B, C \in \mathbb{C}$ komplex ——— m
 $(x-\beta)(x-\bar{\beta}) = x^2+px+q$

rozklad
na parciální
lomky

$$\frac{P(x)}{Q(x)} = \frac{A_1}{(x-\alpha)} + \frac{A_2}{(x-\alpha)^2} + \dots + \frac{A_m}{(x-\alpha)^m} + \frac{B_1x+C_1}{(x^2+px+q)} + \dots + \frac{B_mx+C_m}{(x^2+px+q)^m}$$

pr. $\frac{5x-23}{x^2-8x+7} = \frac{5x-23}{(x-1)(x-7)} = \frac{A}{x-1} + \frac{B}{x-7} = \frac{3}{x-1} + \frac{2}{x-7}$

1) metoda neurčitých koeficientů

$$\frac{5x-23}{(x-1)(x-7)} = \frac{A(x-7)+B(x-1)}{(x-1)(x-7)}$$

$$5x-23 = Ax-7A+Bx-B$$

$$x^0 \quad -23 = -7A - B$$

$$x^1 \quad 5 = A + B$$

$$-18 = -6A$$

$$\rightarrow A=3; B=2$$



2) dosazení kořenů
pro kořeny a násobnosti 1

$$5x - 23 = A(x-7) + B(x-1)$$

$$x=1 \quad 5-23 = -6A + B0$$

$$\underline{A=3}$$

$$x=7 \quad 35-23 = A0 + 6B$$

$$\underline{B=2}$$

metoda zakryvací:

$$A = \frac{5x-23}{x-7} \Big|_{x=1} = 3$$

$$B = \frac{5x-23}{x-1} \Big|_{x=7} = 2$$

pr.

$$\frac{x^4 + 5x^3 + 7x^2 + 7x + 9}{x^2 + 5x + 6} =$$

$$\begin{aligned} (x^4 + 5x^3 + 7x^2 + 7x + 9) : (x^2 + 5x + 6) &= x^2 + 1 + \frac{2x+3}{x^2+5x+6} \\ - (x^4 + 5x^3 + 6x^2) & \\ \hline & \quad x^2 + 7x + 9 \\ & \quad - (x^2 + 5x + 6) \\ & \quad \hline & \quad 2x + 3 \end{aligned}$$

$$= x^2 + 1 + \frac{3}{x+3} - \frac{1}{x+2}$$

$$\frac{2x+3}{x^2+5x+6} = \frac{2x+3}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$2x+3 = A(x+3) + B(x+2)$$

$$x = -2 \quad -1 = A$$

$$x = -3 \quad -3 = -B$$

$$B = 3$$

rozložit

$$\text{př. } \frac{11}{2x^2+9x-5} = \frac{11}{2(x+5)(x-\frac{1}{2})} = \frac{A}{x-\frac{1}{2}} + \frac{B}{x+5}$$

$$-9 \pm \sqrt{81+40} = \frac{-9 \pm 11}{2} \rightarrow -5, \frac{1}{2}$$

pozor! musíme dostat
př. polynom

$$\frac{11}{2} = A(x+5) + B(x-\frac{1}{2})$$

$$x = -5 \quad \frac{11}{2} = -\frac{11}{2}B \quad x = \frac{1}{2} \quad \frac{11}{2} = \frac{11}{2}A$$

$$B = -1 \quad A = 1$$

$$= \frac{1}{x-\frac{1}{2}} - \frac{1}{x+5}$$

$$\bullet \quad \frac{2x^3 - 3x^2 + 4x + 6}{2x^4 + 3x^3} = \frac{2x^3 - 3x^2 + 4x + 6}{2x^3(x + \frac{3}{2})} = \frac{A}{2(x + \frac{3}{2})} + \frac{B}{x^3} + \frac{C}{x^2} + \frac{D}{x}$$

$$2x^3 - 3x^2 + 4x + 6 = Ax^3 + 2B(x + \frac{3}{2}) + 2Cx(x + \frac{3}{2}) + 2Dx^2(x + \frac{3}{2})$$

$$= Ax^3 + 2Bx + 3B + 2Cx^2 + 3Cx + 2Dx^3 + 3Dx^2$$

$$2D + A = 2$$

$$2C + 3D = -3$$

$$2B + 3C = 4$$

$$3B = 6$$

$$B = 2$$

$$C = 0$$

$$D = -1$$

$$A = 4$$

$$\frac{4}{2(x + \frac{3}{2})} + \frac{2}{x^3} - \frac{1}{x}$$

$$\frac{6x^2 + 9x + 17}{x^3 + x^2 + 3x - 5} = \frac{6x^2 + 9x + 17}{(x-1)(x^2 + 2x + 5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+2x+5} =$$

$$6x^2 + 9x + 17 = A(x^2 + 2x + 5) + (Bx + C)(x - 1)$$

$$\underline{A}x^2 + \underline{2A}x + \underline{5A} + \underline{B}x^2 + \underline{C}x - \underline{B}x - \underline{C}$$

$$\begin{aligned} A + B &= 6 \\ 2A - B + C &= 9 \\ 5A - C &= 17 \end{aligned}$$

$$3A + C = 15$$

$$5A - C = 17$$

$$10A = 32$$

$$\underline{A = 4}$$

$$\underline{B = 2}$$

$$\underline{C = 3}$$

$$= \frac{4}{x-1} + \frac{2x+3}{x^2+2x+5}$$

$$\frac{5x^2 - 7x + 12}{x^3 - x^2 + 4x - 4} = \frac{5x^2 - 7x + 12}{(x-1)(x^2 + 4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}$$

$$\begin{array}{rrrr} 1 & -1 & 4 & -4 \\ & 1 & 0 & 4 \\ \hline 1 & 0 & 4 & 0 \end{array}$$

$$A = \frac{5x^2 - 7x + 12}{x^2 + 4} \Big|_{x=1} = \frac{10}{5} = \underline{2}$$

$$Bx + C = \frac{5x^2 - 7x + 12}{x-1} \Big|_{\substack{x=2j \\ x=-2j}}$$

$$2j : 2Bj + C = \frac{-20 - 14j + 12}{2j - 1}$$

$$-2j : -2Bj + C = \frac{-20 + 14j + 12}{-2j - 1}$$

$$2Bj - 4 = \frac{-8 - 14j}{2j - 1}$$

$$2Bj = \frac{-8 - 14j + 8j - 4}{2j - 1} = \frac{-12 - 6j}{-1 + 2j}$$

$$-4B - 2Bj = -12 - 6j$$

$$\underline{B = 3}$$

$$2C = \frac{-20 - 14j + 12}{2j - 1} - \frac{-20 + 14j + 12}{-2j - 1}$$

$$2C = \frac{-16j - 8 + 28j + 12}{-5} = \frac{12 - 10j}{-5}$$

$$= -\frac{40}{5} = \underline{-8}$$

$$\underline{C = -4}$$

partial

$$\frac{12}{x^3+8} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+4} = \frac{1}{x+2} + \frac{-x+1}{x^2-2x+4}$$

$$Q(x) = x^3+8 = (x+2)(x^2-2x+4)$$

$$\begin{array}{r} 1 \ 0 \ 0 \ 8 \\ -2 \ 4 \ -8 \\ \hline 1 \ -2 \ 4 \ 0 \end{array}$$

$$12 = A(x^2-2x+4) + (Bx+C)(x+2)$$

$$12 = Ax^2 - 2Ax + 4A + Bx^2 + 2Bx + Cx + 2C$$

$$x^2: \quad A+B = 0$$

$$B = 0-A$$

$$x^1: -2A + 2B + C = 0$$

$$x^0: \quad 4A + 2C = 12$$

$$4A = 12 - 8$$

$$\underline{A = 1}$$

$$-2A - 2A + C = 0$$

$$\underline{4A + 2C = 12}$$

$$3C = 12$$

$$\underline{C = 4}$$

$$1+B = 0$$

$$\underline{B = -1}$$

$$\frac{-x^3+5x^2+9x-13}{(x^2+5x+6)(x^2+x-2)} = \frac{\quad}{(x+3)(x+2)(x-1)(x+2)}$$

$$= \frac{A}{(x+2)^2} + \frac{B}{x+2} + \frac{C}{x+3} + \frac{D}{x-1}$$

$$-x^3+5x^2+9x-13 = A(x+3)(x-1) + B(x+2)(x+3)(x-1) + C(x+2)^2(x-1) + D(x+2)^2(x+3)$$

$$A(x^2+2x-3) + B(x^3+5x^2+6x-x^2-5x-6) + C(x^3+x^2-4x-4) + D(x^3+5x^2+6x+2x^2+10x+12)$$

$$(Ax^2+2Ax-3A) + (Bx^3+4Bx^2+Bx-6B) + (Cx^3+3Cx^2-4C) + (Dx^3+7Dx^2+16Dx+12D)$$

-11-

$$x^3: -1 = B + C + D$$

$$x^2: 5 = A + 4B + 3C + 7D$$

$$x^1: 9 = 2A + B + \cancel{4C} + 16D$$

$$x^0: -13 = -3A - 6B - 4C + 12D$$

$$B = -1 - C - D$$

$$5 = A - 4 - 4C - 4D + 3C + 7D$$

$$9 = 2A - 1 - C - D + 16D$$

$$-13 = -3A + 6 + 6C + 6D - 4C + 12D$$

$$9 = A - C + 3D$$

$$10 = 2A - C + 15D$$

$$-13 = -3A + 2C + 18D$$

$$A = 9 + C - 3D$$

$$10 = 18 + 2C - 6D - C + 15D$$

$$-19 = -27 - 3C + 9D + 2C + 18D$$

$$-8 = C + 9D$$

$$8 = -C + 27D$$

$$0 = D$$

$$C = -8$$

$$A = 1$$

$$B = 7$$

$$= \frac{1}{(x+2)^2} + \frac{7}{x+2} + \frac{-8}{x+3}$$

Matice

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{pmatrix}$$

- $m = n$... čtvercová mat.
- $A = B (m, m)$; $a_{ij} = b_{ij}$

Matice. kalkulus

1) Součet / rozdíl

$$A = [a_{ij}] \quad (m, n)$$

$$B = [b_{ij}] \quad (m, n)$$

$$C = A \pm B, \quad C = [c_{ij}]$$

$$c_{ij} = a_{ij} \pm b_{ij}$$

2) Násobení reál. číslem λ :

3) Součin

$$A(m, n); B(n, p)$$

$$C = A \cdot B; C(m, p)$$

$$c_{ij} = a_{i1} \cdot b_{1j} + a_{i2} \cdot b_{2j} + \dots + a_{in} \cdot b_{nj} = \sum_{k=1}^n a_{ik} \cdot b_{kj}$$

$$A = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 & -3 \\ 2 & 2 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 7 & 4 & -7 \\ 7 & 8 & 7 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 0 & 1 \end{pmatrix} \quad B = C$$

$$D = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ -1 & -1 & 1 \end{pmatrix}$$

$$E = A \cdot B + C \cdot D$$

$$E = \begin{pmatrix} 5 & 2 & -1 \\ 15 & 6 & -3 \end{pmatrix} + \begin{pmatrix} 8 & 8 & -8 \\ 1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 13 & 10 & -9 \\ 16 & 7 & -4 \end{pmatrix}$$

Vlastnosti

1) asociativnost

$$(A \cdot B) \cdot C = A(B \cdot C)$$

$$(A + B) + C = A + (B + C)$$

2) komutativnost

$$A \cdot B \neq B \cdot A$$

$$A + B = B + A$$

3) distribucionost nás. vzhledem ke sčítání

$$A \cdot (B + C) = AB + AC$$

$$A \cdot 0 = 0$$

$$A \neq 0; B \neq 0 \quad A \cdot B = 0$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

najít B tak, aby $A \cdot B = 0$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$a + c = 0$$

$$a = -c$$

$$b + d = 0$$

$$b = -d$$

$$B = \begin{pmatrix} a & b \\ -a & -b \end{pmatrix}$$

majit A , aby $A^2 = A$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$(1) \quad a^2 + bc = a$$

$$(2) \quad ab + bd = b$$

$$(3) \quad ac + dc = c$$

$$(4) \quad bc + d^2 = d$$

$$b = b(a+d)$$

$$c = c(a+d)$$

$$\textcircled{1} \quad a+d \neq 0$$

$$b=0$$

$$c=0$$

$$a^2 = a$$

$$d^2 = d$$

$$a(a-1) = 0$$

$$d(d-1) = 0$$

$$\underline{a=1 \quad d=1}$$

$$\textcircled{2} \quad a+d=1$$

$$b=b \quad c=c$$

$$(1) \quad (1-d)^2 + bc = 1-d$$

$$(4) \quad bc + d^2 = d$$

$$\underline{1-2d+d^2+bc = 1-d}$$

$$\underline{bc+d^2=d}$$

$$d^2-d+bc=0$$

$$d^2-d+bc=0$$

$$(1) \quad a^2 + bc = d$$

$$(2) \quad \underline{a=1-d}$$

$$\begin{pmatrix} a & \frac{a-a^2}{c} \\ c & 1-a \end{pmatrix} \begin{pmatrix} a & \frac{a-a^2}{c} \\ c & 1-a \end{pmatrix} =$$

$$= \begin{pmatrix} a & \frac{a-a^2}{c} \\ c & 1-a \end{pmatrix}$$

$$a-a^2 = 1-a$$

$$\begin{aligned} a, c \\ d &= 1-a \\ b &= \frac{a-a^2}{c} \end{aligned}$$

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

najít B , aby $A \cdot B = B \cdot A$

$$B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 2a+c & 2b+d \\ -a+3c & -b+3d \end{pmatrix} = B \cdot A = \begin{pmatrix} 2a-b & a+3b \\ 2c-d & c+3d \end{pmatrix}$$

$$2a+c = 2a-b \rightarrow c = -b$$

$$2b+d = a+3b$$

$$(2) \quad -a+3c = 2c-d$$

$$-b+3d = c+3d \rightarrow c = -b$$

$$d = a + b$$

$$d = a - c$$

$$0 = b + c$$

$$(3) \quad 0 = -a + c + d$$

$$a = \alpha$$

$$b = -\gamma$$

$$c = \gamma$$

$$d = \alpha - \gamma$$

$$B = \begin{pmatrix} \alpha & -\gamma \\ \gamma & \alpha - \gamma \end{pmatrix} = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \gamma \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

Inverzní matice

Čtvercová X typu n je inverzní k matici A typu n ,
jestliže platí $A \cdot X = E = X \cdot A$
jednotková matice

Ke čtverci A typu n existuje $A^{-1} \Leftrightarrow A$ je regulární; $\text{hod}(A) = n$

Linear. závislost a nezávislost

Vektory $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_m$, pak jejich lineární nulovou kombinací
 $\alpha_1 \vec{x}_1 + \alpha_2 \vec{x}_2 + \dots + \alpha_m \vec{x}_m = \vec{0}$

Vektory $L \neq Z \Leftrightarrow$ ex. jejich nulová. netriviální lin. komb.

Hledání

$$(A|E) \xrightarrow{\text{řádkové úpravy}} (E|A^{-1})$$

$$\left(\begin{array}{ccc|ccc} 3 & 4 & -8 & 1 & 0 & 0 \\ 1 & 1 & -3 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -3 & 0 & 1 & 0 \\ 3 & 4 & -8 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 1 & -3 & 0 & 1 & 0 \\ 0 & 4 & -5 & 1 & 0 & -3 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & -3 & 0 & 1 & 0 \\ 0 & 4 & -5 & 1 & 0 & -3 \\ 0 & 1 & -2 & 0 & 1 & -1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & -3 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & -3 & 0 \\ 0 & 0 & -3 & -1 & 4 & -1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & +\frac{1}{3} & -\frac{4}{3} & \frac{4}{3} \\ 0 & 1 & 0 & +\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{3} & -\frac{4}{3} & \frac{1}{3} \end{array} \right) ?$$

nicht inv. m.

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & 1 & 0 \\ -1 & 2 & 4 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ -1 & 2 & 4 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 & 1 & 0 \end{array} \right) =$$

$$= \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 6 & 1 & 0 & 1 \\ 0 & 0 & 3 & 0 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{1}{3} & 0 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{3} & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{1}{3} & 0 \end{array} \right)$$

$$\underline{A^{-1} = \frac{1}{3} \begin{pmatrix} 2 & 0 & -1 \\ 1 & -2 & 1 \\ 0 & 1 & 0 \end{pmatrix}}$$

DÜ $A = \begin{pmatrix} 3 & -1 & 3 \\ -2 & 2 & 0 \\ -1 & 3 & 3 \end{pmatrix}$; $A^{-1} = ?$

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 3 & 1 & 0 & 0 \\ -2 & 2 & 0 & 0 & 1 & 0 \\ -1 & 3 & 3 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 4 & -4 & 0 & 1 & 0 & -1 \\ 0 & -4 & -6 & 0 & 1 & -2 \\ -1 & 3 & 3 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & -4 & -6 & 0 & 1 & -2 \\ -1 & 3 & 3 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & -4 & -6 & 0 & 1 & -2 \\ 0 & 2 & 3 & \frac{1}{4} & 0 & \frac{3}{4} \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{array} \right)$$

$$\boxed{A \cdot X = B} ; X = ? ; A \dots \text{je regulární}$$

$$(A|B) \xrightarrow{\text{řádk. úpravy}} (E|X)$$

$$\begin{array}{c} A \qquad B \\ \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 9 & 9 & 9 \\ 2 & 1 & -2 & 0 & 0 & 0 \\ 2 & -2 & 1 & 0 & 0 & 0 \end{array} \right) \text{ viz tabulka} \end{array}$$

$$A \cdot X = B$$

$$\begin{array}{c} (A) \qquad (B) \\ \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 3 & 2 & 0 & 0 \\ -1 & -2 & 4 & 0 & 3 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 1 & 2 \\ 0 & -1 & 6 & 0 & 4 & 3 \\ 0 & 0 & 1 & 2/3 & 0 & 0 \end{array} \right) \sim \\ \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -4/3 & 1 & 2 \\ 0 & -1 & 0 & -2 & 4 & 3 \\ 0 & 0 & 1 & 2/3 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -16/3 & 5 & 5 \\ 0 & 1 & 0 & 4 & -4 & -3 \\ 0 & 0 & 1 & 2/3 & 0 & 0 \end{array} \right) \\ X = \begin{pmatrix} -16/3 & 5 & 5 \\ 4 & -4 & -3 \\ 2/3 & 0 & 0 \end{pmatrix} \end{array}$$

DÚ 2

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} ; B = \begin{pmatrix} 6 & 3 \\ 12 & 6 \end{pmatrix} ; \underline{A \cdot X = B}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 12 & 6 \end{pmatrix}$$

není
regulární

$$A = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} ; B = \begin{pmatrix} 3 & 5 \\ -1 & -2 \end{pmatrix} ; C = \begin{pmatrix} 14 & 9 \\ 16 & 10 \end{pmatrix}$$

$$A \cdot X \cdot B = C \quad | \cdot A^{-1}$$

$$\underbrace{A^{-1} \cdot A}_{E} \cdot X \cdot B = A^{-1} \cdot C \quad | \cdot B^{-1}$$

$$\underbrace{X \cdot B \cdot B^{-1}}_E = A^{-1} \cdot C \cdot B^{-1} \quad \boxed{\text{pozor na poradi!}}$$

$$X = A^{-1} \cdot C \cdot B^{-1}$$

$$\begin{pmatrix} 5 & 7 & | & 1 & 0 \\ 6 & 8 & | & 0 & 1 \end{pmatrix} \xrightarrow{I_1 - I_2} \sim \begin{pmatrix} 5 & 7 & | & 1 & 0 \\ 0 & 2 & | & 6 & -5 \end{pmatrix} \sim \begin{pmatrix} 5 & 7 & | & 1 & 0 \\ 0 & 1 & | & 3 & -\frac{5}{2} \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 5 & 0 & | & -20 & \frac{35}{2} \\ 0 & 1 & | & 3 & -\frac{5}{2} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & -4 & \frac{7}{2} \\ 0 & 1 & | & 3 & -\frac{5}{2} \end{pmatrix} \quad \underline{A^{-1} = \begin{pmatrix} -4 & \frac{7}{2} \\ 3 & -\frac{5}{2} \end{pmatrix}}$$

$$\begin{pmatrix} 3 & 5 & | & 1 & 0 \\ -1 & -2 & | & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{5}{3} & | & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & | & \frac{1}{3} & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & | & 2 & 5 \\ 0 & 1 & | & -1 & -3 \end{pmatrix} \quad \underline{B^{-1} = \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix}}$$

$$\begin{pmatrix} -4 & \frac{7}{2} \\ 3 & -\frac{5}{2} \end{pmatrix} \cdot \begin{pmatrix} 14 & 9 \\ 16 & 10 \end{pmatrix} \cdot B^{-1} = \begin{pmatrix} 0 & -1 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}}}$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 0 \\ 2 & 3 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A \cdot X \cdot A = B \cdot A^2 \quad | \cdot A^{-1}$$

$$X \cdot A = A^{-1} \cdot B \cdot A \cdot A \quad | \cdot A^{-1}$$

$$X = A^{-1} \cdot B \cdot A$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 2 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & -3 & 0 & 2 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & 1 \\ 0 & 1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array} \right)$$

$$A^{-1} = \begin{pmatrix} -2 & -1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & -1 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} -2 & 0 & 0 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ -1 & -1 & 0 \\ 2 & 3 & 2 \end{pmatrix} = \begin{pmatrix} -2 & -4 & -2 \\ 2 & 4 & 2 \\ -1 & -2 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}$$

$$A - X \cdot A = E_3$$

$$X \cdot A = A - E_3 \quad | \cdot A^{-1}$$

$$X = A \cdot A^{-1} - E_3 \cdot A^{-1}$$

$$X = E - A^{-1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & +1 & +2 & 0 & -1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array} \right)$$

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 & 2 & -1 \\ 1 & 0 & 0 \\ -1 & -1 & 2 \end{pmatrix}}}$$

$$A = \begin{pmatrix} 0 & 7 & 0 \\ 8 & 0 & 3 \\ 5 & 0 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 10 & 0 & 3 \\ 0 & 6 & 0 \\ 0 & 3 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A \cdot X = B \cdot X + C$$

$$AX - BX = C$$

$$(A-B)X = C \quad | \cdot (A-B)^{-1}$$

$$X = (A-B)^{-1}C$$

$$A-B = \begin{pmatrix} -10 & 7 & -3 \\ 8 & -6 & 3 \\ 5 & -3 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} -10 & 7 & -3 & 1 & 0 & 0 \\ 8 & -6 & 3 & 0 & 1 & 0 \\ 5 & -3 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} -2 & 1 & 0 & 1 & 1 & 0 \\ -7 & 3 & 0 & 0 & 1 & -3 \\ 5 & -3 & 1 & 0 & 0 & 1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} -2 & 1 & 0 & 1 & 1 & 0 \\ -3 & 1 & 0 & -2 & -1 & -3 \\ -2 & 0 & 1 & 0 & 1 & -2 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 3 \\ -3 & 1 & 0 & -2 & -1 & -3 \\ -2 & 0 & 1 & 0 & 1 & -2 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 2 & 3 \\ 0 & 1 & 0 & 7 & 5 & 6 \\ 0 & 0 & 1 & 6 & 5 & 4 \end{array} \right)$$

$$X = \begin{pmatrix} 3 & 2 & 3 \\ 7 & 5 & 6 \\ 6 & 5 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & 3 \\ 7 & -5 & 6 \\ 6 & -5 & 4 \end{pmatrix}$$

Determinant

Def: Bud' A číselná matice typu n

Číslo $\sum_{\pi=(i_1, i_2, \dots, i_n)} \text{sgn } \pi \cdot a_{1i_1} \cdot a_{2i_2} \cdot \dots \cdot a_{ni_n}$ nazýváme $\det A$

(2,2)

$$\det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

(3,3) Sarrusovo pravidlo

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Klasifikace

A regulární, $\det A \neq 0$

A singularní, $\det A = 0$

$$A = \begin{pmatrix} \triangle \\ 0 \end{pmatrix}; \det A = a_{11} \cdot a_{22} \cdot \dots \cdot a_{nn}$$

$$\det A^{-1} = \frac{1}{\det A}$$

$$\det A^T = \det A$$

nás radek - det je taky
prohoz ř. - -det

$$\det \begin{pmatrix} 3 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & -1 \end{pmatrix} = -12$$

$$3 \cdot 0 \cdot 1 - 3 \cdot 0$$

$$1 \cdot 2 \cdot 1 - 1 \cdot 2$$

$$1 \cdot 2 \cdot (-1) - 1 \cdot 2$$

$$-6 + 0 + 2 - 2 - 6 + 0$$

= -12

převést na (S)

$$\begin{vmatrix} 3 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & -1 \end{vmatrix} \xrightarrow{(-\frac{1}{3})} \begin{vmatrix} 3 & 0 & 1 \\ 0 & 2 & \frac{2}{3} \\ 0 & 0 & -2 \end{vmatrix} = -12$$

upravovaný řádek nesmíme násobit, aby
to vyšlo

$$\begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & -2 \end{vmatrix} =$$

$$= \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & -4 \end{vmatrix} = - \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -4 \end{vmatrix} = -16$$

$$\begin{vmatrix} 3 & -1 & 4 & 2 \\ 1 & 3 & 2 & -3 \\ -2 & 2 & -3 & -1 \\ 1 & -3 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 4 & 2 \\ 0 & 6 & 1 & -6 \\ 0 & -4 & -1 & 5 \\ 1 & -3 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 6 & 1 & -6 \\ 0 & -4 & -1 & 5 \\ 3 & -1 & 4 & 2 \end{vmatrix} =$$

$$= - \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 6 & 1 & -6 \\ 0 & -4 & -1 & 5 \\ 0 & 8 & 1 & -7 \end{vmatrix} = + \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 6 & 1 & -6 \\ 0 & 1 & 1 & -7 \\ 0 & -4 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 6 & 1 & -6 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -1 \end{vmatrix} =$$

$$= - \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 3 \\ 0 & 6 & 1 & -6 \end{vmatrix} = - \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 1 & -3 \end{vmatrix} =$$

$$= - \begin{vmatrix} 1 & -3 & 1 & 3 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{vmatrix} = \underline{0}$$

$$\begin{vmatrix} m+1 & 1 & 1 & 1 \\ 1 & m+1 & 1 & 1 \\ 1 & 1 & m+1 & 1 \\ 1 & 1 & 1 & m+1 \end{vmatrix} \xrightarrow{\text{od 2-4 odečteme 1.}} = \begin{vmatrix} m+1 & 1 & 1 & 1 \\ -m & m & 0 & 0 \\ -m & 0 & m & 0 \\ -m & 0 & 0 & m \end{vmatrix} =$$

b.1. sloupce
přičteme všechny
ostatní

$$= \begin{vmatrix} m+4 & 1 & 1 & 1 \\ 0 & m & 0 & 0 \\ 0 & 0 & m & 0 \\ 0 & 0 & 0 & m \end{vmatrix} = \underline{m^3(m+4)}$$

matice řádu n

$$\begin{vmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2-x & 1 & 1 & \dots & 1 \\ 1 & 1 & 3-x & \dots & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ m & 1 & 1 & 1 & \dots & m-x \end{vmatrix}$$

$$= 0, \quad x = ?$$

od každého odečteme 1. řádek

$$= \begin{vmatrix} 1 & 1 & 1 & 1 & \dots & 1 \\ 0 & 1-x & 0 & 0 & \dots & 0 \\ 0 & 0 & 2-x & 0 & \dots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (m-1)-x & 0 \end{vmatrix}$$

$$= (1-x)(2-x) \dots (m-1-x) = 0$$

$$\underline{x \in \{1, 2, 3, \dots, m-1\}}$$

od každého ř. odečteme n -tý

$$\begin{vmatrix}
 1 & n & n & \dots & n \\
 n & 2 & n & \dots & n \\
 n & n & 3 & \dots & n \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 n & n & n & \dots & n
 \end{vmatrix}
 =
 \begin{vmatrix}
 1-n & 0 & 0 & \dots & 0 \\
 0 & 2-n & 0 & \dots & 0 \\
 0 & 0 & 3-n & \dots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & 0 & \dots & n
 \end{vmatrix}
 =
 \underbrace{(1-n)(2-n)(3-n)\dots n}_{n!}
 = (-1)^{n-1} n(n-1)(n-2)\dots =$$

$$= \underline{\underline{(-1)^{n-1} \cdot n!}}$$

od každého ř. odečteme 1. k 1. ř. přičteme vše ostat.

$$\begin{vmatrix}
 n & 1 & 1 & \dots & 1 \\
 1 & n & 1 & \dots & 1 \\
 1 & 1 & n & \dots & 1 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 1 & 1 & 1 & \dots & n
 \end{vmatrix}
 =
 \begin{vmatrix}
 n & 1 & 1 & \dots & 1 \\
 1-n & n-1 & 0 & \dots & 0 \\
 1-n & 0 & n-1 & \dots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 1-n & 0 & 0 & \dots & n-1
 \end{vmatrix}
 =
 \begin{vmatrix}
 2n-1 & 1 & 1 & \dots & 1 \\
 0 & n-1 & 0 & \dots & 0 \\
 0 & 0 & n-1 & \dots & 0 \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & 0 & \dots & n-1
 \end{vmatrix}$$

$$= \underline{\underline{(n-1)^{n-1} \cdot (2n-1)}}$$

Rozvoj determinantu

Doplňek D_{ij} matice A příslušný pozici (i, j) je číslo $D_{ij} = (-1)^{i+j}$ det A_{ij}

Věta A matice typu (n, n) , $n \geq 2$, det $A = a_{i1}D_{i1} + a_{i2}D_{i2} + \dots + a_{in}D_{in}$

pomocí rozvoje

$$\begin{vmatrix} 3 & 0 & 2 \\ 1 & 3 & 5 \\ -4 & 2 & 0 \end{vmatrix} = 3 \begin{vmatrix} 3 & 5 \\ 2 & 0 \end{vmatrix} + 2 \begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix} = -30 + 28 = \underline{-2}$$

$$\begin{vmatrix} 1 & 2 & 4 & -1 \\ 2 & 1 & 2 & 2 \\ 1 & 3 & 1 & 2 \\ 2 & 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} + 4 \begin{vmatrix} 2 & 1 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{vmatrix} =$$

$$= \cancel{12} \cancel{12} (17-12) - 2(14-44) + 4(12-17) + 0 =$$

$$= 5 + 0 - 20 = \underline{-15}$$

Nechť A typu (n, n)

1) A singular $\rightarrow A^{-1}$ neexist.

2) A regulár. $\rightarrow A^{-1}$ existuje a platí $A^{-1} = \frac{1}{\det A} [D_{ij}]^T$

najít takto inverzi

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

$$\det A = \underline{-4}$$

$$D = \begin{pmatrix} 1 & +6 & -2 \\ +1 & 2 & -2 \\ -3 & -6 & 2 \end{pmatrix}$$

$$D^T = \begin{pmatrix} 1 & +1 & -3 \\ +6 & 2 & -6 \\ -2 & -6 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \\ -\frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 4 & -8 \\ 1 & 1 & -3 \\ 1 & 0 & -1 \end{pmatrix}$$

$$\det A = -15 + 12 = -3$$

$$D = \begin{pmatrix} -1 & -2 & -1 \\ 4 & 5 & 4 \\ -4 & 1 & -1 \end{pmatrix}$$

$$D^T = \begin{pmatrix} -1 & 4 & -4 \\ -2 & 5 & 1 \\ -1 & 4 & -1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{4}{3} & \frac{4}{3} \\ \frac{2}{3} & -\frac{5}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{4}{3} & \frac{1}{3} \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 3 \\ -1 & 2 & 4 \end{pmatrix}$$

$$\det A = -3 - 6 = -9$$

$$D = \begin{pmatrix} -6 & -3 & 0 \\ 0 & 6 & -3 \\ 3 & -3 & 0 \end{pmatrix}$$

$$D^T = \begin{pmatrix} -6 & 0 & 3 \\ -3 & 6 & -3 \\ 0 & -3 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -\frac{2}{3} & 0 & -\frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

Lineární závislost, nezávislost

Lineární kombinace

$x_1, x_2, \dots, x_m$. Lineární kombinací těchto vektorů je

je vektor $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_m x_m$; $\alpha_1, \alpha_2, \dots, \alpha_m \in \mathbb{R}$

Př je $(3, 10, 1)$ lin. kombinací ^{-těchto v?} $(3, 1, 0)$, $(-3, 5, 4)$, $(0, -1, 1)$. ?

$$\alpha_1(3, 1, 0) + \alpha_2(-3, 5, 4) + \alpha_3(0, -1, 1) = (3, 10, 1)$$

$$3\alpha_1 - 3\alpha_2 = 3$$

$$\alpha_1 + 5\alpha_2 - \alpha_3 = 10$$

$$4\alpha_2 + \alpha_3 = 1$$

$$\left(\begin{array}{ccc|c} 3 & -3 & 0 & 3 \\ 1 & 5 & -1 & 10 \\ 0 & 4 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 1 & 5 & -1 & 10 \\ 0 & 4 & 1 & 1 \end{array} \right) \sim$$

~~$\alpha_1 = 2$
 $10\alpha_2 = 10$
 $\alpha_2 = 1$
 $\alpha_3 = -3$~~

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 6 & -1 & 9 \\ 0 & 4 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 10 & 0 & 10 \\ 0 & 4 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 4 & 1 & 1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -3 \end{array} \right)$$

$$\alpha_1 = 2$$

$$\alpha_2 = 1$$

$$\alpha_3 = -3$$

je lin. kombinací, neboť existují $\alpha_1, \alpha_2, \alpha_3$,
alespoň 1 nenulový

if $(1, -15, 6)$ lin. h. $(2, -2, 3), (0, 1, 0), (1, 11, -3)$?

$$\alpha_1 (2, -2, 3) + \alpha_2 (0, 1, 0) + \alpha_3 (1, 11, -3) = (1, -15, 6)$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ -2 & 1 & 11 & -15 \\ 3 & 0 & -3 & 6 \end{array} \right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ -2 & 1 & 11 & -15 \\ 1 & 0 & -1 & 2 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 9 & -11 \\ +1 & 0 & -1 & +2 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & 9 & -11 \\ 0 & 0 & -3 & +3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -20 \\ 0 & 0 & 1 & -1 \end{array} \right)$$

$$\alpha_1 = 3$$

$$\alpha_2 = -2$$

$$\alpha_3 = -1$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -1 & +1 \end{array} \right) ? \rightarrow$$

$$\alpha_1 = 1$$

$$\alpha_2 = -2$$

$$\alpha_3 = -1$$

$(4, 6, 0)$ comb. $(2, 3, 5), (2, 4, 7), (0, -1, -2)$

$$\left(\begin{array}{ccc|c} 2 & 2 & 0 & 4 \\ 3 & 4 & -1 & 6 \\ 5 & 7 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 3 & 4 & -1 & 6 \\ 5 & 7 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & -1 & 6 \\ 0 & 2 & -2 & -10 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & -1 & 6 \\ 0 & 1 & -1 & -5 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 0 & -11 \end{array} \right) \sim$$

half. rel. $\rightarrow$ semi lin. comb.

Lineární závislost skupiny

Def: $x_1, x_2, \dots, x_n$ nazveme LZ, jestliže existuje její nulová netrivi.
lin. kombinace $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = \vec{0}$

pro LZ? $(2, 1, -3, 0), (1, 3, -1, 1), (1, 1, -3, 0)$
pro LN

$(2, 0, 1), (1, -1, 0), (3, 1, 1)$ pro LZ?

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 2 & 0 \\ 0 & -1 & -1 & 0 \\ 1 & 0 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\alpha = -t$$

$$\beta = -t$$

$$\gamma = t$$

pro LZ

určit h , aby $(2, 1, 3), (1, 2, -5), (3, 0, h)$ byly LZ

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 2 & -5 & 0 \\ 3 & -5 & h & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 8 & 0 \\ 0 & 3 & - & 0 \\ 0 & -11 & h & 0 \end{array} \right)$$

$$h = 11$$

najdi a , aby $(a, 1, 0, 1), (0, 2, 2, 2), (-1, 2, -1, 0), (2, 0, 1, a)$ byly LZ

$$\begin{pmatrix} a & 0 & -1 & 2 & | & 0 \\ 1 & 2 & 2 & 0 & | & 0 \\ 0 & 2 & -1 & 1 & | & 0 \\ 1 & 2 & 0 & a & | & 0 \end{pmatrix} \sim \begin{pmatrix} a & 0 & -1 & 2 & | & 0 \\ 1 & 0 & 3 & -1 & | & 0 \\ 0 & 2 & -1 & 1 & | & 0 \\ 0 & 0 & -2 & a & | & 0 \end{pmatrix} \sim \begin{pmatrix} a & 0 & -1 & 2 & | & 0 \\ 1 & 0 & 3 & -1 & | & 0 \\ 0 & 2 & -1 & 1 & | & 0 \\ 0 & 0 & -2 & a & | & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} x & y & z & w & | & 0 \\ 1 & 0 & 3 & -1 & | & 0 \\ 0 & 0 & (-3a-1) & (a+2) & | & 0 \\ 0 & 2 & -1 & 1 & | & 0 \\ 0 & 0 & -2 & a & | & 0 \end{pmatrix} \sim$$

$$(3) \quad (-3a-1)y + (a+2)z = 0$$

$$(4) \quad -2y + az = 0$$

$$y = \frac{az}{2}$$

$$(-3a-1)\left(\frac{az}{2}\right) + (a+2)z = 0$$

$$z \left(\frac{-3a^2 - a}{2} + a + 2 \right) = 0$$

$$-3a^2 - a + 2a + 4 = 0$$

$$3a^2 - a - 4 = 0$$

$$a_2 = \frac{1 \pm \sqrt{1+48}}{6} =$$

$$\begin{matrix} \frac{4}{3} \\ -1 \end{matrix}$$

from polynomials $x^3 + 2x^2 + x - 2$, $3x^3 + 8x^2 + 2x - 3$, $x^3 + 4x^2 + x + 2$,
 $2x^3 + 6x^2$ LZ/LN ?

$$\begin{pmatrix} 1 & 3 & 1 & 2 & | & 0 \\ 2 & 8 & 4 & 6 & | & 0 \\ 1 & 2 & 1 & 0 & | & 0 \\ -2 & -3 & 2 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 1 & 2 & | & 0 \\ 0 & 5 & 6 & 6 & | & 0 \\ 0 & -1 & 0 & -2 & | & 0 \\ 0 & 3 & 4 & 4 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -3 & -2 & | & 0 \\ 0 & 1 & 0 & 2 & | & 0 \\ 0 & 5 & 6 & 6 & | & 0 \\ 0 & 3 & 4 & 4 & | & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & -3 & -2 & | & 0 \\ 0 & 1 & 0 & 2 & | & 0 \\ 0 & 0 & 3 & -2 & | & 0 \\ 0 & 0 & 4 & -2 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & -4 & | & 0 \\ 0 & 1 & 0 & 2 & | & 0 \\ 0 & 0 & -1 & 0 & | & 0 \\ 0 & 0 & 0 & -2 & | & 0 \end{pmatrix}$$

$\alpha = 0$
 $\beta = 0$
 $\gamma = 0$
 $\delta = 0$

from lin. mixărilor

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 \\ 3 & -1 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 4 \\ 4 & 2 \end{pmatrix} \quad D = \begin{pmatrix} 9 & -16 \\ 11 & 1 \end{pmatrix}$$

if D linear combination A, B, C ?

$$\begin{pmatrix} 2 & 1 & 0 & | & 9 \\ -1 & -2 & 4 & | & -16 \\ 0 & 3 & 4 & | & 11 \\ 4 & -1 & 2 & | & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 4 & | & -7 \\ -1 & -2 & 4 & | & -16 \\ 0 & 3 & 4 & | & 11 \\ 0 & -3 & 2 & | & -17 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 4 & | & -7 \\ 0 & -3 & 8 & | & -23 \\ 0 & 3 & 4 & | & 11 \\ 0 & 0 & 6 & | & -6 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -4 & 0 & | & -18 \\ 0 & -3 & 8 & | & -23 \\ 0 & 0 & 12 & | & -12 \\ 0 & 0 & 6 & | & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & -4 & 0 & | & -18 \\ 0 & -3 & 8 & | & -23 \\ 0 & 0 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -4 & 0 & | & -18 \\ 0 & -3 & 0 & | & -15 \\ 0 & 0 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$\gamma = -1 \quad \beta = 5 \quad \alpha = 2$

D is lin. combination

u prostorech $\mathbb{R}^3$

$$f(x) = \sin^2 x \quad x \in \mathbb{R}$$

$$g(x) = 3 \cos^2 x$$

$$h(x) = 4$$

je-li na nich def. norm L^2/L^∞ ?

$$a \sin^2 x + 3b \cos^2 x + 4c = 0$$

rovnice platí pro

$$x = \frac{\pi}{2}: a + 4c = 0$$

$$a = 1$$

$$x = -\frac{\pi}{2}: a + 4c = 0$$

$$b = \frac{1}{3}$$

$$x = 0: 3b + 4c = 0$$

$$c = -\frac{1}{4}$$

$$-a + 3b = 0$$

$$3b = a$$

je-li L^2

je-li $f(x) = \sin x$

$g(x) = |\cos x|$

$h(x) = \cos(x + \frac{\pi}{4})$

L^2/NZ na $(-\frac{\pi}{2}, \frac{\pi}{2})$?

$$x = 0: a \sin x + b |\cos x| + c \cos(x + \frac{\pi}{4}) = 0$$

$$b + \frac{\sqrt{2}}{2} c = 0$$

$$x = \frac{\pi}{4}: a \frac{\sqrt{2}}{2} + b \frac{\sqrt{2}}{2} = 0$$

$$x = -\frac{\pi}{4}: -\frac{\sqrt{2}}{2} a + b \frac{\sqrt{2}}{2} + c = 0$$

$$\left(\begin{array}{ccc|c} 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \sqrt{2} & 1 & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \sqrt{2} & 1 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & \sqrt{2} & 1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

preto $L \subset \mathbb{R}^3$

b) na $(-\pi; \pi)$ - kvázi abs. hodnota $\angle N$

Lineárny podpriestor

Def: Množina lineár. priestoru L

Nepriárodnou množinou M , $M \subseteq L$ nazývame podpriestorom L ,

jestliže $\forall x, y \in M$: (1) $x + y \in M$
(2) $\alpha \cdot x \in M$

Lineárny obal

L ; lin. obal $x_1, x_2, \dots, x_n$ je množina lineár. kombinácií $x_1, x_2, \dots, x_n$
 $\langle x_1, x_2, \dots, x_n \rangle$

Báze

Báze L je taková podmnožina $B \subseteq L$:

(1) $\langle B \rangle = L$

(2) B je l.n.

Dimenzia L $\dim L$

Dimenzia L je počet prvků báze

$\forall L, M \subseteq L$. Potom $\dim M \leq \dim L$

Veta o dimenziách

Nechť M, N jsou podpriestory L , pak

$$\dim M + \dim N = \dim(M \cup N) + \dim(M \cap N)$$

musí mít bázi

Pr

$$2x_1 - x_2 + x_3 = 0$$

$$x_1 + x_2 - x_3 = 0$$

$$4x_1 + x_2 - x_3 = 0$$

M je mn. všech řešení soustavy

1) je M podprostorem L ?

2) určit $\dim M$

3) najít nějakou bázi

$$\left(\begin{array}{ccc|c} 2 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 4 & 1 & -1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & -1 & 1 & 0 \\ 4 & 1 & -1 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_1 = 0$$

$$x_3 = t, \quad t \in \mathbb{R}$$

$$x_2 = t$$

$$x = (0, t, t)$$

$$\left. \begin{array}{l} (0, 1, 1) \\ (0, 2, 2) \\ (0, 3, 3) \end{array} \right\} M$$

M je lin. podprostor $\mathbb{R}^3$

báze $M = \{(0, 2, 2)\}$ a samozřejmě jeden libovolný vektor $\in M$

$$\dim M = 1$$

Př.

$$x_1 - x_2 + 2x_3 = 0$$

$$2x_1 - 2x_2 + 4x_3 = 0$$

$$-x_1 + x_2 - 2x_3 = 0$$

stejně starý

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 2 & -2 & 4 & 0 \\ -1 & 1 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_1 = t, \quad t, x \in \mathbb{R}$$

$$x_3 = x$$

$$x_2 = 2x + t$$

$$\underline{x = (t, 2x+t, x)}$$

$$\left. \begin{array}{l} t=0, x=1 \quad (0, 2, 1) \\ 1 \quad 0 \quad (1, 1, 0) \\ 1 \quad 1 \quad (1, 3, 1) \\ 2 \quad 1 \quad (2, 4, 1) \\ \vdots \end{array} \right\} M$$

M lin podprostor $\mathbb{R}^3$ ✓ $M \subseteq \mathbb{R}^3$

v písemce samozřejmě zdůvodnit :)

dim bude max 3, zvolíme nejake 3 prvky :

$$\text{hod} \begin{pmatrix} 0 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} = 2 = \dim M$$

báze jsou libovolné 2 vektory

$$\text{báze } M = \{(0, 2, 1), (1, 1, 0)\}$$

Pr

$$a = (1, 1, 0, 1)$$

$$b = (1, 0, 1, 1)$$

$$c = (5, 3, 2, 5)$$

$$u = (1, 0, 0, 0)$$

$$v = (4, 1, 1, 2)$$

$$M = \langle a, b, c \rangle$$

$$V = \langle u, v \rangle$$

najedle $\dim M = 2$

$$\dim V = 2$$

$$\dim M \cup V = 3$$

$$\dim M \cap V = 1, \text{ k velky 0 dimenzich}$$

$$\dim M = \text{hod} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 5 & 3 & 2 & 5 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & -2 & 2 & 0 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 2$$

$$\dim V = \text{hod} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 2 \end{pmatrix} = 2$$

synajeme

$$\begin{aligned} \dim(M \cup V) &= \text{hod} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 5 & 3 & 2 & 5 \\ 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 2 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 2 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} \\ &= \text{hod} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = 3 \end{aligned}$$

dalsi $M = \langle a, b \rangle$

$$V = \langle u, v \rangle$$

najit mn. $Z : Z \in M \cap V$

$$Z = (z_1, z_2, z_3, z_4) = \alpha a + \beta b = \gamma u + \delta v$$

$$\alpha \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \gamma \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 4 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & -4 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & 1 & 0 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 1 & 1 & -1 & -4 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -3 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & -3 & -1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2 & -1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 & -1 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \text{rank} = 3$$

$$\delta = t \quad t \in \mathbb{R}$$

$$\gamma = -2t$$

$$\beta = t$$

$$\alpha = t$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = t \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad t \in \mathbb{R}$$

$$= -2t \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 1 \\ 2 \end{pmatrix} = t \begin{pmatrix} 2 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

Ru.

$$a = (2, 1, 0, 3)$$

$$b = (1, 1, -1, 2)$$

$$c = (1, 0, 1, 1)$$

$$u = (0, 2, -2, 1)$$

$$v = (1, 3, 0, 2)$$

$$w = (2, 4, 2, 3)$$

$$M = \langle a, b, c \rangle$$

$$V = \langle u, v, w \rangle$$

najít bázi $M \cap V$

?

$$Z = M \cap V$$

$$x = (x_1, x_2, x_3, x_4) = \alpha a + \beta b + \gamma c = \delta u + \epsilon v + \vartheta w$$

$$\begin{pmatrix} 2 & 1 & 1 & 0 & -1 & -2 \\ 1 & 1 & 0 & -2 & -3 & -4 \\ 0 & -1 & 1 & 2 & 0 & -2 \\ 3 & 2 & 1 & -1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 2 & 2 & 2 \\ 1 & 1 & 0 & -2 & -3 & -4 \\ 0 & -1 & 1 & 2 & 0 & -2 \\ 0 & -1 & 1 & 5 & 7 & 9 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 0 & 1 & 2 & 2 & 2 \\ 0 & -1 & -1 & 0 & -5 & -6 \\ 0 & -1 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 3 & 7 & 11 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 2 & 2 & 2 \\ 0 & -1 & -1 & 0 & -5 & -6 \\ 0 & 0 & 0 & 2 & -5 & -8 \\ 0 & 0 & 0 & 3 & 7 & 11 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & 2 & 2 & 2 \\ 0 & -1 & -1 & 0 & -5 & -6 \\ 0 & 0 & 0 & 6 & -15 & -24 \\ 0 & 0 & 0 & 6 & -14 & -22 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1 & 2 & 2 & 2 \\ 0 & -1 & -1 & 0 & -5 & -6 \\ 0 & 0 & 0 & 6 & -15 & -24 \\ 0 & 0 & 0 & 0 & -29 & -46 \end{pmatrix} \quad ?$$

$$\sim \begin{pmatrix} 2 & 1 & 1 & 0 & -1 & -2 \\ 0 & -1 & 1 & 4 & 5 & 6 \\ 0 & 0 & 0 & 2 & 5 & 8 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

$$\vartheta = s$$

$$\epsilon = -2s$$

$$2\delta - 10s + 8s = 0$$

$$\delta = s$$

$$\gamma = t: -\beta + t + 4s - 12s + 6s = 0$$

$$\beta = t$$

$$2\alpha + t + t + 2s - 2s = 0$$

$$\alpha = -t$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = -t \begin{pmatrix} 2 \\ 1 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= s \begin{pmatrix} 0 \\ 2 \\ -2 \\ 1 \end{pmatrix} - 2s \begin{pmatrix} 1 \\ 3 \\ 0 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$M \cap V$ je triviální prostor - nemá bázi

• $a = (1, -1, 2, -2)$

$b = (2, 0, 1, 3)$

$c = (0, 2, -3, 1)$

$u = (2, 1, -1, 2)$

$v = (1, 1, 1, 1)$

$w = (1, 0, -2, 1)$

$M = \langle a, b, c \rangle$

$V = \langle u, v, w \rangle$

najít bázi $M \cap V$

$$\begin{pmatrix} 1 & 2 & 0 & -2 & -1 & -1 \\ -1 & 0 & 2 & -1 & -1 & 0 \\ 2 & 1 & -3 & 1 & -1 & 2 \\ -2 & 3 & 1 & -2 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 & -2 & -1 & -1 \\ 0 & 2 & 2 & -3 & -2 & -1 \\ 0 & -3 & -3 & -3 & 1 & 2 \\ 0 & 7 & 1 & -6 & -3 & -3 \end{pmatrix} \begin{matrix} \\ \cdot 1.3 \\ \cdot 1.2 \end{matrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 0 & -2 & -1 & -1 \\ 0 & 2 & 2 & -3 & -2 & -1 \\ 0 & 0 & 0 & -15 & -4 & 1 \\ 0 & 7 & 1 & -6 & -3 & -3 \end{pmatrix} \begin{matrix} \\ \cdot 1.2 \\ \cdot 1.2 \end{matrix} \rightarrow \begin{pmatrix} 1 & 2 & 0 & -2 & -1 & -1 \\ 0 & 2 & 2 & -3 & -2 & -1 \\ 0 & 0 & 12 & -9 & -8 & -1 \\ 0 & 0 & 0 & -15 & -4 & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & -2 & 1 & 1 & 0 \\ 0 & 2 & 2 & -3 & -2 & -1 \\ 0 & 0 & 12 & -9 & -8 & -1 \\ 0 & 0 & 0 & -15 & -4 & 1 \end{pmatrix} \quad \begin{matrix} \text{můž. vysl.} \\ \text{ověřit} \end{matrix}$$

$$\begin{pmatrix} x & \beta & \gamma & \delta & \varepsilon & \vartheta \\ 1 & 2 & 0 & -2 & -1 & -1 \\ 0 & 2 & 2 & -3 & -2 & -1 \\ 0 & 0 & -6 & 5 & 2 & 3 \\ 0 & 0 & 0 & -1 & 4 & -5 \end{pmatrix}$$

$$\varepsilon = t, \quad \vartheta = s, \quad t, s \in \mathbb{R}$$

$$\underline{\delta = 4t - 5s}$$

$$6\gamma = 5(4t - 5s) + 2t + 3s = 22t - 22s$$

$$\underline{\gamma = \frac{11}{3}(t - s)}$$

$$-2\beta = 2 \cdot \frac{11}{3}(t - s) - 3(4t - 5s) + 2t + s =$$

$$= \frac{22}{3}(t - s) - 12t + 15s + 2t + s = \left(\frac{22}{3} - 10\right)(t - s) = -\frac{8}{3}(t - s)$$

$$\underline{\beta = \frac{4}{3}(t - s)}$$

$$-\alpha = \frac{20}{3}(t - s) - 2(4t - 5s) - t - s = \frac{20}{3}(t - s) - 9t + 9s =$$

$$= (t - s) \left(\frac{20}{3} - 9\right) = -\frac{7}{3}(t - s)$$

$$\underline{\alpha = \frac{7}{3}(t - s)}$$

$$\mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = (4t - 5s) \begin{pmatrix} 2 \\ 1 \\ -1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ -2 \\ 1 \end{pmatrix} = t \begin{pmatrix} 9 \\ 5 \\ -3 \\ 9 \end{pmatrix} + s \begin{pmatrix} -9 \\ -5 \\ 3 \\ -9 \end{pmatrix}$$

prüfstand
base

$$\begin{pmatrix} 9 \\ 5 \\ -3 \\ 9 \end{pmatrix}$$

MAN

1. 2. 3. 4.

DÚ

$$a = (2, 0, 1, 1)$$

$$b = (3, 2, 1, -1)$$

$$c = (1, 2, 0, -2)$$

$$u = (1, 3, -1, 2)$$

$$v = (2, 1, 1, 0)$$

$$w = (1, 2, 2, 2)$$

$$M = \langle a, b, c \rangle$$

$$V = \langle u, v, w \rangle$$

Bázi $M \cap V$

$$a = (2, 3, 5)$$

$$u = (7, -2, a), a \in \mathbb{R}$$

$$b = (3, 7, 8)$$

$$M = \langle a, b, c \rangle$$

$$c = (-1, -6, 1)$$

$\dim M$?

a ? aby $u \in M$

$$\dim M = \text{hod} \begin{pmatrix} 2 & 3 & 5 \\ 3 & 7 & 8 \\ 1 & -6 & 1 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & -6 & 1 \\ 0 & 15 & 3 \\ 0 & 25 & 5 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & -6 & 1 \\ 0 & 5 & 1 \\ 0 & 5 & 1 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & -6 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \underline{\underline{2}}$$

$$\begin{pmatrix} 2 & 3 & 5 \\ 3 & 7 & 8 \\ 1 & -6 & 1 \\ 7 & -2 & a \end{pmatrix} \xrightarrow{(1 \leftrightarrow 3)} \begin{pmatrix} 1 & -6 & 1 \\ 0 & 0 & 0 \\ 0 & 5 & 1 \\ 0 & 40 & a-7 \end{pmatrix} \sim \begin{pmatrix} 1 & -6 & 1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \\ 0 & 5 & \frac{a-7}{5} \end{pmatrix}$$

$$\frac{a-7}{5} = 1$$

$$\underline{\underline{a = 12}}$$

vektor u nemá směřovat hodnost

pr. $\text{hod} \begin{pmatrix} 5 & a & -1 & 2 \\ 2 & -1 & a & 1 \\ 5 & 3 & -1 & 2 \end{pmatrix} =$

najít a , aby měla matice
co nejmenší hodnotu.

$$= \text{hod} \begin{pmatrix} 5 & a & -1 & 2 \\ 2 & -1 & a & 1 \\ 0 & a-3 & 0 & 0 \end{pmatrix}$$

2

$$\text{hod} \begin{pmatrix} a & 4 & 6 & 1 \\ 3 & 1 & 0 & 4 \\ -1 & 1 & 2 & -1 \\ 2 & 2 & 2 & 3 \end{pmatrix}$$

... *nejmenší*

... *matice*

$\rightarrow a=0, \text{hod} = ?$
 $\rightarrow a \neq 0, \text{hod} = 3$

$$\text{hod} \begin{pmatrix} a & a & 1 \\ a & 1 & a \\ 1 & a & a \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{pmatrix} \xrightarrow{(-a)} \text{hod} \begin{pmatrix} 1 & a & a \\ 0 & 1-a^2 & a^2-a^2 \\ 0 & a-a^2 & 1-a^2 \end{pmatrix} =$$

$$= \text{hod} \begin{pmatrix} 1 & a & a \\ 0 & (a+1)(a-1) & a(a-1) \\ 0 & a(a-1) & (a+1)(a-1) \end{pmatrix}$$

$\rightarrow a=1, \text{hod} = 1$
 $\rightarrow a \neq 1$

$$\text{hod} \begin{pmatrix} 1 & a & a \\ 0 & a+1 & a \\ 0 & a & a+1 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & a & a \\ 0 & 1 & -1 \\ 0 & a & a+1 \end{pmatrix} \xrightarrow{(-a)} =$$

$$= \text{hod} \begin{pmatrix} 1 & a & a \\ 0 & 1 & -1 \\ 0 & 0 & 2a+1 \end{pmatrix}$$

$\rightarrow a=1, \text{hod} = 1$
 $\rightarrow a \neq 1$
 $\rightarrow a = -\frac{1}{2}, \text{hod} = 2$
 $\rightarrow a \neq -\frac{1}{2}, \text{hod} = 3$

Souřadnice vektoru v uspořádané bázi

Def: Je-li $e_1, e_2, \dots, e_m$ báze V , potom uspoř. m-tici $(e_1, e_2, \dots, e_m)$ nazýváme uspoř. bázi.

Je-li $x \in V$, pak $x = x_1 e_1 + x_2 e_2 + \dots + x_m e_m$

Arithm. vektoru $(x_1, x_2, \dots, x_m)$ říkáme souřadnice x vzhledem k bázi E

Pr $(3, 0, 1); (1, 2, 1); (1, 2, -1)$ je báze B v $\mathbb{R}^3$;

souřadnice $x = (5, 4, -3)$ vzhledem k B

$$x_1 \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 1 & | & 5 \\ 0 & 2 & 2 & | & 4 \\ 1 & 1 & -1 & | & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & | & -3 \\ 0 & 2 & 2 & | & 4 \\ 0 & -2 & 4 & | & 14 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 & | & -3 \\ 0 & 1 & 1 & | & 2 \\ 0 & 0 & 4 & | & 3 \end{pmatrix} \sim$$
$$\sim \begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 3 \end{pmatrix} \quad \begin{array}{l} x = 1 \\ y = -1 \\ z = 3 \end{array}$$

souřadnice x vzhledem k B jsou $(1, -1, 3)$

$E = (e_1, e_2, e_3)$ je báze v $\mathbb{R}^3$

vektor $(1, 5, 2)$ je souř. vektorem a vyjádřím ho $\in E$

$F = (e_1 + 2e_2, e_2 + 2e_3, e_1 + 2e_3)$? je báze v $\mathbb{R}^3$?

Když ano, najít souř. k F

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & -2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 6 \end{pmatrix}$$

nebo díky $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 1 & 0 & 2 \end{vmatrix} = 4 \Rightarrow$ je regulární

$\rightarrow F$ je báze v $\mathbb{R}^3$

$$x = e_1 + 5e_2 + 2e_3 = y_1 f_1 + y_2 f_2 + y_3 f_3$$

$$= y_1 (e_1 + 2e_2) + y_2 (e_2 + 2e_3) + y_3 (e_1 + 2e_3)$$

$$= e_1 (\underbrace{y_1 + y_3}_1) + e_2 (\underbrace{2y_1 + y_2}_5) + e_3 (\underbrace{2y_2 + 2y_3}_2)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 5 \\ 0 & 2 & 2 & 2 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 1 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & -2 \end{array} \right)$$

~~$$\begin{aligned} y_3 &= -\frac{4}{3} \\ y_2 &= \frac{7}{3} \\ y_1 &= \frac{2}{3} \end{aligned}$$~~

Získáme

$$\left(\frac{5}{3}, \frac{5}{3}, -\frac{2}{3} \right)$$

na písemku 2 list bílý
nadepsaný

datum č. poslucháče	varianta	jm. a příjmení průběh č. úlohy 202 - M. Piskáček
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Pr $E = (e_1, e_2, e_3)$ je báze v $\mathbb{R}^3$

$(12, 9, 6)$ souřadice vektoru x vzhledem k E

$$x = (12, 9, 6)_E$$

$$F = (f_1, f_2, f_3) = (2e_1 + e_2 + 2e_3, e_1 - e_2 + e_3, 3e_1 + 2e_2 + e_3)$$

je také báze v $\mathbb{R}^3$, uveď souř. x v F

musí
být
regulární

$$\begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 3 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 5 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

můžeme
i přes determinant...

$$x = 12e_1 + 9e_2 + 6e_3 = y_1 f_1 + y_2 f_2 + y_3 f_3$$

$$= y_1(2e_1 + e_2 + 2e_3) + y_2(e_1 - e_2 + e_3) + y_3(3e_1 + 2e_2 + e_3)$$

$$= e_1(2y_1 + y_2 + 3y_3) + e_2(y_1 - y_2 + 2y_3) + e_3(2y_1 + y_2 + y_3)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 12 \\ 1 & -1 & 2 & 9 \\ 2 & 1 & 1 & 6 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -1 & -6 \\ 0 & 3 & -3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -1 & -6 \\ 0 & 0 & -2 & -6 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

$$x = (2, -1, 3)_F$$

$$P(x) = x^3 + 5x^2 + 6x + 5$$

najít souř. vzhledem k bázi

$$B = (b_1, b_2, b_3, b_4) = (1, x+1, (x+1)^2, (x+1)^3) =$$

$$= (1, x+1, x^2+2x+1, x^3+3x^2+3x+1)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 3 & 6 \\ 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right)$$

$$x_4 = 1$$

$$x_3 = 2$$

$$x_2 = -1$$

$$x_1 = 3$$

$$(P(x))_B = (3, -1, 2, 1)$$

Lineární rovnice

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = c_1$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = c_m$$

nebo $A \cdot x^T = c^T$; $A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}$; $x^T = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$; $c^T = \begin{pmatrix} c_1 \\ \vdots \\ c_m \end{pmatrix}$

Frobeniova v.

soust. má řešení, když $\text{hod } \bar{A} = \text{hod } A$

$\hookrightarrow \text{hod } \bar{A} = \text{hod } A \rightarrow \text{řes. existuje}$
 $\text{hod } \bar{A} = \text{hod } A+1 \rightarrow \text{nemá řes.}$

Q

① Homogenní soustavy

$$A \cdot x^T = 0^T; c = (0, 0, \dots, 0)$$

vždy alespoň 1 řešení

a) $\left(\begin{array}{ccc|c} \diagup & & & 0 \\ 0 & \diagup & & 0 \\ & & \diagup & \vdots \\ 0 & \dots & 0 & 0 \end{array} \right) \rightarrow \infty \text{ mnoho řešení}$
parametry

b) $\left(\begin{array}{ccc|c} \diagup & & & 0 \\ 0 & \diagup & & 0 \\ & & \diagup & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \rightarrow 1 \text{ řešení}$
triviální

pr $x_1 - 3x_2 - x_3 + 5x_4 = 0$

$$2x_1 + x_2 + 5x_3 + 3x_4 = 0$$

$$x_1 + x_2 + 3x_3 + x_4 = 0$$

$$\left(\begin{array}{cccc|c} 1 & -3 & -1 & 5 & 0 \\ 2 & 1 & 5 & 3 & 0 \\ 1 & 1 & 3 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & -3 & -1 & 5 & 0 \\ 0 & 7 & 7 & -7 & 0 \\ 0 & 4 & 4 & -4 & 0 \end{array} \right) \sim$$

$$\left(\begin{array}{cccc} 1 & -3 & -1 & 5 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_4 = \lambda$$

$$x_3 = \lambda$$

$$\lambda, \lambda \in \mathbb{R}$$

$$x_2 = \lambda - \lambda$$

$$x_1 = -2\lambda - 2\lambda$$

$$x = (x_1, x_2, x_3, x_4) = (-2\lambda - 2\lambda, \lambda - \lambda, \lambda, \lambda) =$$

$$= \lambda (-2, -1, 1, 0) + \lambda (-2, 1, 0, 1)$$

sakale ^{řekne} kapsa

$$x_1 + 2x_2 + 4x_3 - 3x_4 = 0$$

$$3x_1 + 5x_2 + 6x_3 - 4x_4 = 0$$

$$4x_1 + 5x_2 - 2x_3 + 3x_4 = 0$$

$$3x_1 + 8x_2 + 24x_3 - 19x_4 = 0$$

$$\begin{pmatrix} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 4 & 5 & -2 & 3 \\ 3 & 8 & 24 & -19 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & -7 & -6 & 15 \\ 0 & 2 & 12 & -10 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_4 = \lambda$$

$$x_3 = \lambda \quad \lambda, \lambda \in \mathbb{R}$$

$$x_2 = -6\lambda + 5\lambda$$

$$x_1 = 8\lambda - 7\lambda$$

$$x = \lambda(8, -6, 1, 0) + \lambda(-7, 5, 0, 1)$$

② Nehomogenní soustava

$$Ax^T = c^T$$

a) $\left(\begin{array}{cc|c} \diagup & & c_1 \\ 0 & \diagup & c_2 \\ \hline 0 & 0 & 0 \end{array} \right)$

∞ mnoho řes.
parametrů

b) $\left(\begin{array}{cc|c} \diagup & & c_1 \\ 0 & \diagup & c_2 \\ \hline 0 & 0 & 0 \end{array} \right)$

právě 1 netriviální

c) $\left(\begin{array}{cc|c} \diagup & & c_1 \\ 0 & \diagup & c_2 \\ \hline 0 & 0 & c \neq 0 \end{array} \right)$

nená řes.

$$2x - z = 3$$

$$-x + y + 2z = 1$$

$$x + y + z = 2$$

$$\left(\begin{array}{ccc|c} 2 & 0 & -1 & 3 \\ -1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 3 & 3 \\ 0 & -2 & -3 & -1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{array} \right)$$

nená řešení

$$2x + y + z = 1$$

$$5x - 2y - 5z = 1$$

$$4x - y - 3z = 1$$

$$x - y - 2z = 0$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 5 & -2 & -5 & 1 \\ 4 & -1 & -3 & 1 \\ 1 & -1 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 3 & 5 & 1 \\ 0 & 3 & 5 & 1 \\ 0 & 3 & 5 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -1 & -2 & 0 \\ 0 & 3 & 5 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$z = \lambda \quad \lambda \in \mathbb{R}$$

$$y = \frac{1-5\lambda}{3}$$

$$x = \frac{1-5\lambda}{3} + 2\lambda = \frac{\lambda+1}{3}$$

$$\star \left(\frac{\lambda+1}{3}, \frac{1-5\lambda}{3}, \lambda \right) = \lambda \left(\frac{1}{3}, -\frac{5}{3}, 1 \right) + \left(\frac{1}{3}, \frac{1}{3}, 0 \right)$$

③ Nehomogenní soust. s regulární matricí

$$Ax^T = c^T, \quad A \dots \text{regulární}$$

$$\left(\begin{array}{l} A \cdot x^T = 0^T \rightarrow \text{triviální řešení} \\ \rightarrow \text{právě 1 netriviální řeš. } u^T = A^{-1} \cdot c^T \end{array} \right.$$

Cramerovo pravidlo

$$Ax^T = b^T, \quad u = (u_1, u_2, \dots, u_n)$$

$$u_i = \frac{\det B_i}{\det A}$$

pr. vypočítejte x

$$2x - 4y + 3z = 1$$

$$x - 2y + 4z = 3$$

$$3x - y + 5z = 2$$

$$\det A = \begin{vmatrix} 2 & -4 & 3 \\ 1 & -2 & 4 \\ 3 & -1 & 5 \end{vmatrix} = \cancel{-20} - \cancel{48} - 3 + \cancel{48} + \cancel{20} + \cancel{4} - 30 + 5 = \underline{-25}$$

$$\det B_1 = \begin{vmatrix} 1 & -4 & 3 \\ 3 & -2 & 4 \\ 2 & -1 & 5 \end{vmatrix} = \cancel{-10} - \cancel{32} - 9 + \cancel{42} + \cancel{60} + 4 - 20 + 50 = \underline{25}$$

$$x = \frac{25}{-25} = \underline{-1}$$

pr.

$$\begin{aligned}x + y - z &= a \\ 2x - y + z &= b \\ x + 4y - 4z &= c\end{aligned}$$

1) na jakých podmínkách má řešení? nebo vyřešit

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & a \\ 2 & -1 & 1 & b \\ 1 & 4 & -4 & c \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & a \\ 0 & -3 & 3 & b-2a \\ 0 & 3 & -3 & c-a \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & a \\ 0 & -3 & 3 & b-2a \\ 0 & 0 & 0 & c-3a+b \end{array} \right)$$

$3a - b - c \neq 0 \dots$ soustava nemá řešení

$$3a - b - c = 0 :$$

$$\underline{z = s}$$

~~$$y = \frac{b-2a}{3} + s$$~~

$$y = \frac{b-2a}{3} + s$$

$$x = -\left(s - \frac{b-2a}{3}\right) + s + a$$

$$x = \frac{-b + 2a}{3} + s$$

na tabuli:

$$M = \left(\frac{4a+c}{3} ; \frac{c-a}{3} ; 0 \right) + \langle (0, 1, 1) \rangle$$

$$\begin{aligned}x_1 + x_2 - px_3 &= 1 \\ x_1 - 2x_2 + 3x_3 &= 2 \\ x_1 + px_2 - x_3 &= 1\end{aligned}$$

řešíš

1) pro jaké p je regul./singul

~~$$\left(\begin{array}{ccc|c} 1 & 1 & -p & 1 \\ 1 & -2 & 3 & 2 \\ 1 & p & -1 & 1 \end{array} \right)$$~~

$$\begin{vmatrix} 1 & 1 & -p \\ 1 & -2 & 3 \\ 1 & p & -1 \end{vmatrix} = \begin{vmatrix} 2 & 3 - p^2 - 2p - 3p + 1 \\ & & \end{vmatrix} = -p^2 - 5p + 6$$

$$p^2 + 5p - 6 = 0$$

$$(p-1)(p+6) = 0$$

a) $p \neq 1 \wedge p \neq -6$

$$x = \frac{1}{\det A} \cdot \begin{vmatrix} 1 & 1 & -1 \\ 2 & -2 & 3 \\ 1 & p & -1 \end{vmatrix} = \frac{2+3-\cancel{2p^2}-2p-3p+2}{-(p-1)(p+6)} = \frac{-2p^2-5p+7}{-(p-1)(p+6)} =$$

$$= \frac{2p^2+5p-7}{(p-1)(p+6)} = \frac{\cancel{(p-1)}(2p+7)}{\cancel{(p-1)}(p+6)} = \frac{2p+7}{p+6}$$

$$y = \frac{1}{\det A} \cdot \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 3 \\ 1 & 1 & -1 \end{vmatrix} = \frac{-2+3-\cancel{p}+2p-3+1}{-(p-1)(p+6)} = \frac{p-1}{-(p-1)(p+6)} = \frac{-1}{p+6}$$

$$z = \frac{1}{\det A} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & p & 1 \end{vmatrix} = \frac{-2+1+p+2-2p-1}{-(p-1)(p+6)} = \frac{-(p-1)}{-(p-1)(p+6)} = \frac{1}{p+6}$$

b) $p = 1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 1 & -2 & 3 & 2 \\ 1 & 1 & -1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -3 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\underline{z = 3t} \quad -3y + 12t = 1$$

$$\underline{y = 4t - \frac{1}{3}}$$

$$x + 4t - \frac{1}{3} - 3t = 1$$

$$\underline{x = -t + \frac{4}{3}}$$

$$M = \left(\frac{4}{3}, -\frac{1}{3}, 0 \right) + t(-1, 4, 3)$$

$$\underline{M = \left(\frac{4}{3}, -\frac{1}{3}, 0 \right) + \langle (-1, 4, 3) \rangle}$$

c) $p = -6$

$$\left(\begin{array}{ccc|c} 1 & 1 & +6 & 1 \\ 1 & -2 & 3 & 2 \\ 1 & -6 & -1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -6 & 1 \\ 0 & -3 & -3 & 1 \\ 0 & -7 & -7 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -6 & 1 \\ 0 & -3 & -3 & 1 \\ 0 & 0 & 0 & -5 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -6 & 1 \\ 0 & -3 & -3 & 1 \\ 0 & 0 & 0 & -7 \end{array} \right)$$

pro $p = -6$ nemá řešení

řešte soustavu

$$x_1 + 2x_2 + 2x_4 = 1$$

$$2x_1 + 3x_2 + 2x_3 + x_4 = 2$$

$$x_1 + 3x_2 - 2x_3 + px_4 = 1$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 1 \\ 2 & 3 & 2 & 1 & 2 \\ 1 & 3 & -2 & p & 1 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 1 \\ 0 & -1 & 2 & -3 & 0 \\ 0 & 1 & -2 & p-2 & 0 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 2 & 1 \\ 0 & -1 & 2 & -3 & 0 \\ 0 & 0 & 0 & p-5 & 0 \end{array} \right)$$

a) $p = 5$

$$x_4 = \alpha \quad x_3 = \lambda \quad \alpha, \lambda \in \mathbb{R}$$

$$x_2 = 2\lambda - 3\alpha$$

$$x_1 = 1 - 4\lambda + 6\alpha - 2\alpha = 1 - 4\lambda + 4\alpha$$

$$M = (1, 0, 0, 0) + \langle (-4, 2, 1, 0), (4, -3, 0, 1) \rangle$$

$$b) p \neq 5$$

$$x_4 = 0 \quad x_3 = 0 \quad x_2 = 20$$

$$\underline{a \in \mathbb{R}} \quad = \quad =$$

$$x_1 = -40 + 1$$

$$M = (1, 0, 0, 0) + \langle (-4, 2, 1, 0) \rangle$$

Matice přechodu

Def: Necht $B = (b_1, \dots, b_m)$, $C = (c_1, \dots, c_n)$ uspoř. báze v L .

Matice A , pro kterou platí $(b_1, \dots, b_m) \cdot A = (c_1, \dots, c_n)$
nazveme maticí přechodu $A_{(B,C)}$

Věta $B, C \in L$, $A_{(B,C)}$. Pokud souřadnice $x \in L$

$$x = (x_1, \dots, x_m)_B = (y_1, \dots, y_n)_C$$

$$A_{(B,C)} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix}$$

Standardní báze

$$a \in \mathbb{R}^n, \alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$$

$$(x_1, \dots, x_m)_{(S)}, \quad (S) = ((1, 0, \dots, 0), (0, 1, 0, \dots, 0), \dots, (0, 0, \dots, 1))$$

Věta $B = (b_1, \dots, b_m) \in L$;

$$\text{pak } A_{S,B} = (b_1^T, \dots, b_m^T)$$

keřta $B = (b_1, \dots, b_m)$, $C = (c_1, \dots, c_m)$ uspoř. báze v V

$$(A_{C,B}|E) \sim (b_1^T, \dots, b_m^T | c_1^T, \dots, c_m^T) \sim (E | A_{B,C})$$

• pří

$$E = \{1, x+1, (x+1)^2\} \quad x^2 + 2x + 1$$

$$F = \{1, x-1, (x-1)^2\} \quad x^2 - 2x + 1$$

pon do uspoř. báze?

hdyž ano - mat. přechodu $A_{E,F}$

vyjádřit $P(x) = 5 - 4x + 3x^2$ v obou bázích

1)

$$E: \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \text{ je báze}$$

$$F: \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \text{ je báze}$$

2)

$$\begin{pmatrix} 1 & 0 & 1 & | & 0 & 0 & 1 \\ 0 & 1 & 2 & | & 0 & 1 & -2 \\ 1 & 1 & 1 & | & 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & | & 1 & 1 & 1 \\ 0 & 1 & 2 & | & 0 & 1 & -2 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & | & 1 & 0 & 2 \\ 0 & 1 & 0 & | & 0 & 1 & -2 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{pmatrix} \xrightarrow{-2} \begin{pmatrix} 1 & 0 & 0 & | & 1 & 0 & 2 \\ 0 & 1 & 0 & | & 0 & 1 & -2 \\ 0 & 0 & 1 & | & 0 & 0 & 1 \end{pmatrix}$$

hde mám
pase chybný!

$$3) \quad P(x) = 5 - 4x + 3x^2 = \alpha \cdot 1 + \beta(x-1) + \gamma(x^2 - 2x + 1)$$

$$x^2: \quad 3 = \gamma$$

$$x: \quad -4 = \beta - 2\gamma \rightarrow \beta = 2$$

$$x^0: \quad 5 = \alpha - \beta + \gamma \rightarrow \alpha = 4$$

$$A_{EF} \cdot \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}_F = \begin{pmatrix} \delta \\ \epsilon \\ \eta \end{pmatrix}_E$$

$$\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ -10 \\ 3 \end{pmatrix}$$

$$P(x) = 12 - 10(x+1) + 3(x+1)^2 = 4 + 2(x-1) + 3(x-1)^2$$

• E, F usporád baze v $\mathbb{R}^3$

$$u = (x_1, x_2, x_3)_E \quad E = (e_1, e_2, e_3)$$

$$u = (y_1, y_2, y_3)_F \quad F = (f_1, f_2, f_3)$$

$$y_1 = 2x_1$$

$$y_2 = 2x_1 + 2x_2$$

$$A_{EF} = ?$$

$$y_3 = 4x_1 + 4x_2 + x_3$$

$$u = x_1 e_1 + x_2 e_2 + x_3 e_3 = y_1 f_1 + y_2 f_2 + y_3 f_3$$

$$= 2x_1 f_1 + 2x_1 f_2 + 2x_2 f_2 + 4x_1 f_3 + 4x_2 f_3 + x_3 f_3$$

$$= x_1 (\underbrace{2f_1 + 2f_2 + 4f_3}_{e_1}) + x_2 (\underbrace{2f_2 + 4f_3}_{e_2}) + x_3 f_3$$

f std. báze (rovnice)

$$f_1 = (1, 0, 0)$$

$$e_1 = (2, 2, 4)$$

$$f_2 = (0, 1, 0)$$

$$e_2 = (0, 4, 4)$$

$$f_3 = (0, 0, 1)$$

$$e_3 = (0, 0, 1)$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 4 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 4 & 1 & -2 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -4 & 1 \end{array} \right)$$

$$A_{E \leftarrow F} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -4 & 1 \end{pmatrix}$$

pr.

$\mathbb{R}^4$

$$e_1 = (1, 1, 1, 1)$$

$$f_1 = (1, 0, 3, 3)$$

$$e_2 = (1, 2, 1, 1)$$

$$f_2 = (1, 2, 1, 1)$$

$$e_3 = (1, 1, 2, 1)$$

$$f_3 = (2, 2, 5, 4)$$

$$e_4 = (1, 3, 2, 3)$$

$$f_4 = (-2, -3, -4, -4)$$

grou po pri báse

$A_{E \leftarrow F}$

$A_{F \leftarrow E}$

$$x = (0, 1, 1, 0)$$

zjistit souř v obou bázích

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 2 & -2 \\ 1 & 2 & 1 & 3 & 0 & 2 & 2 & -3 \\ 1 & 1 & 2 & 2 & 3 & 1 & 5 & -4 \\ 1 & 1 & 1 & 3 & 3 & 1 & 4 & -4 \end{array} \right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 1 & 2 & -2 \\ 0 & 1 & 0 & 2 & -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 & 0 & 3 & -2 \\ 0 & 0 & 0 & 2 & 1 & 0 & 2 & -1 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & +2 & -1 & +1 & -1 \\ 0 & 1 & 0 & 0 & -3 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & -1 \end{array} \right) \sim$$

$$A_{E \leftarrow F} = \begin{pmatrix} 2 & -1 & 1 & -1 \\ -3 & 1 & -2 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 0 & 1 & -1 \end{pmatrix}$$

$$A_{F \leftarrow E} = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 2 & 1 & 1 & -2 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 1 & -3 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 2 & -1 & 1 & -1 & 1 & 0 & 0 & 0 \\ -3 & 1 & -2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right) \sim \left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & -2 & -2 & +2 & 3 & 1 & 0 & -3 \\ 0 & 1 & 2 & -1 & -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & -1 & 0 & 0 & 2 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & +1 & 0 & 0 & -2 & -1 & -1 & +2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & -1 & +1 & +1 & 0 & 0 & -2 \end{array} \right) \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & -1 & 1 \\ 0 & 1 & 0 & 0 & -2 & -1 & -1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & -1 \end{array} \right)$$

$$(0, 1, 1, 0) = \lambda(1, 1, 1, 1) + \beta \dots$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 3 & 1 \\ 1 & 1 & 2 & 2 & 1 \\ -1 & 1 & 1 & 3 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right)$$

$$A_{(E)} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}$$

$$(y_1, y_2, y_3, y_4) = \underline{\underline{(-2, -2, 1, -1)}}$$

Lineární zobrazení

$$B: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$B(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, x_1 - x_2 + 5x_3, x_1 + x_2 + x_3)$$

$$B[(x_1, x_2, x_3) + (y_1, y_2, y_3)] = B(x_1 + y_1, x_2 + y_2, x_3 + y_3) = \dots \text{ and}$$

$$B(\alpha(x_1, x_2, x_3)) = \alpha \cdot (x_1 + 2x_2 + 3x_3, x_1 - x_2 + 5x_3, x_1 + x_2 + x_3)$$

Jádro lin. obraz

Nechť L_1, L_2 lineární prostory

$$0_2 \in L_2 \text{ a } A: L_1 \rightarrow L_2$$

$$\text{Množina } \ker A = \{x \in L_1, A(x) = 0_2\}$$

Pr. $A: \mathbb{R}^3 \rightarrow \mathbb{R}^2$

$$A(1,1,0) \rightarrow (1,-1)$$

$$A(1,0,1) \rightarrow (2,3)$$

$$A(0,1,1) \rightarrow (-1,0)$$

$$x \in \mathbb{R}^3? \quad A(x) = (1,1)$$

$$e_1 = (1,1,0); \quad e_2 = (1,0,1); \quad e_3 = (0,1,1) \text{ jsou úsporní báze v } \mathbb{R}^3$$

$$x = x_1 e_1 + x_2 e_2 + x_3 e_3$$

$$A(x) = A(x_1 e_1 + x_2 e_2 + x_3 e_3) = x_1 A(e_1) + x_2 A(e_2) + x_3 A(e_3) =$$

$$= x_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x_1 + 2x_2 - x_3 = 1$$

$$-x_1 + 3x_2 = 1$$

$$\underline{x_2 = t; \quad t \in \mathbb{R}}$$

$$\underline{x_1 = 3t - 1}$$

$$3t - 1 + 2t - 1 = x_3$$

$$\underline{x_3 = 5t - 2}$$

$$(x_1, x_2, x_3) = (-1, 0, -2) + t(3, 1, 5)$$

$$x = (3t-1) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + (5t-2) \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\underline{x = (-1, -3, -2) + t(4, 8, 6)}$$

$$\underline{\{x, x = (-1, -3, -2) + t(4, 8, 6), t \in \mathbb{R}\}}$$

např. $A(-1, -3, -2) = (1, 1)$

$$\ker A = \{u; u = t(2, 4, 3), t \in \mathbb{R}\}$$

$$\dim A = 1$$

$$\text{hod } A = \dim A(L.) = \text{hod} \begin{pmatrix} 1 & -1 \\ 2 & 3 \\ -1 & 0 \end{pmatrix} = \text{hod} \begin{pmatrix} 1 & -1 \\ 0 & -5 \\ 0 & -1 \end{pmatrix} = \underline{2}$$

příměta

1. rovnice rovnice $\Rightarrow$ param
 2. lin. zobrazení
-

Pr. $A: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$A(4, 11) \Rightarrow ?$ stanovíš hodnotu obrazu

$A(2, 3) \Rightarrow (-1, 2, 1)$

$A(1, -1) \Rightarrow (0, 3, 2)$

$e_1 = (2, 3)$

$e_2 = (1, -1)$ $\Rightarrow$ tvoří bázi $\mathbb{R}^2$

$x = x_1 e_1 + x_2 e_2$

$A(x) = x_1 A(e_1) + x_2 A(e_2)$

$\begin{pmatrix} 4 \\ 11 \end{pmatrix} = x_1 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\left(\begin{array}{cc|c} 2 & 1 & 4 \\ 3 & -1 & 11 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & \frac{1}{2} & 2 \\ 0 & -\frac{5}{2} & 5 \end{array} \right)$

$x_2 = -2$

$x_1 = 3$

$A(4, 11) = 3 \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ -1 \end{pmatrix}$

$$L_1 : A : \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$(0, 1, 2) \in \ker A$$

$$(0, 2, 1) \in \ker A$$

$$A(1, 1, 1) = (2, 1, -3)$$

$$A(2, 1, 3) = ?$$

$$\text{rank } A = ?$$

$$\text{def } A = ?$$

$$A(0, 1, 2) = (0, 0, 0)$$

$$A(0, 2, 1) = (0, 0, 0)$$

$$A(1, 1, 1) = (2, 1, -3)$$

$$f_1 = (0, 1, 2)$$

$$f_2 = (0, 2, 1)$$

$$f_3 = (1, 1, 1)$$

je baze u $\mathbb{R}^3$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & 2 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & -3 \end{array} \right)$$

$$A(x) = x_1 A(f_1) + x_2 A(f_2) + x_3 A(f_3) =$$

$$= x_1 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} =$$

slučaj x_3

$$A(2, 1, 3) = 2 \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}}}$$

$$\text{def } A = 2$$

$$\text{rank } A = \dim A(L_1) = \text{rank} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 1 & -3 \\ 4 & 2 & -6 \end{pmatrix} = \underline{\underline{1}}$$

Ex.

$$a: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$$a(1,1,0) = (1,1)$$

$$a(1,0,1) = (1,-1)$$

$$a(0,1,1) = (-1,-5)$$

$$e_1 = (1,1,0)$$

$$e_2 = (1,0,1)$$

$$e_3 = (0,1,1)$$

doiri baxi

Opisite ker a

$$x = x_1 e_1 + x_2 e_2 + x_3 e_3$$

$$a(x) = x_1 a(e_1) + x_2 a(e_2) + x_3 a(e_3) = (0,0)$$

$$x_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ -1 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -1 & -1 & 0 \\ 1 & -1 & -5 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -2 & -4 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right)$$

$$x_3 = t, \quad t \in \mathbb{R}$$

$$x_2 = -2t$$

$$x_1 = 3t$$

$$x = 3t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - 2t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = t \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$

$$\ker a = \{ x, x = t(1,4,-1); t \in \mathbb{R} \}$$

Matrice lin. schimbătoare

Def: L_1, L_2 lin. prostory, $A: L_1 \rightarrow L_2$ e liniară

$\text{Nech } (B) = (b_1, \dots, b_m)$ baze L_1 , $(C) = (c_1, \dots, c_n)$ baze L_2

Matrice (m, n) , care reprezintă $(a(b_1), a(b_2), \dots, a(b_m)) = (c_1, c_2, \dots, c_n) \cdot A$
A numim matrice lin. schimb.

V. $\forall x \in L_1$; $x = (x_1, x_2, \dots, x_m)_{(B)}$ plată pr. son $a(x) = (y_1, \dots, y_n)_{(C)}$
nouă $A \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$

Ex.

$$A: \mathbb{R}^4 \rightarrow \mathbb{R}^3$$

$$A(1, 1, -1, 0) = (0, 0, 0)$$

$$A(1, 2, -1, -2) = (-1, -3, 1)$$

$$A(1, 0, 0, -1) = (0, 0, 0)$$

$$A(1, 1, 1, 1) = (5, 8, 2)$$

găsiți matrice l. schimb.

găsiți înecna x , $a(x) = \begin{pmatrix} 21 \\ 13 \\ 5 \end{pmatrix}$

$$A \begin{pmatrix} x_1 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \quad A \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A \cdot \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ -1 & -1 & 0 & 1 \\ 0 & -2 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 & 5 \\ 0 & -3 & 0 & 8 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & | & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & | & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & | & 0 & 0 & 1 & 0 \\ 0 & -2 & -1 & 1 & | & 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & | & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & | & 1 & 0 & 1 & 0 \\ 0 & -2 & -1 & 1 & | & 0 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 & 1 & | & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 2 & | & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & | & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 5 & | & 0 & 2 & 0 & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 1 & | & 2 & 3 & 0 & 2 \\ 0 & 0 & 0 & 4 & | & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 4 & | & 1 & 2 & 1 & 1 \end{pmatrix} \times \sim \begin{pmatrix} 1 & 0 & 0 & 0 & | & 3/4 & -1/4 & -5/4 & 3/4 \\ 0 & 1 & 0 & 0 & | & 5/4 & -4/4 & 1/4 & -2/4 \\ 0 & 0 & 1 & 0 & | & 1/4 & 2/4 & 3/4 & 1/4 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & -1 & 0 & 5 \\ 0 & -3 & 0 & 8 \\ 0 & 1 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 & -5 & 3 \\ -2 & 3 & 1 & -2 \\ 5 & -4 & 1 & -2 \\ 1 & 2 & 3 & 1 \end{pmatrix} \cdot \frac{1}{7} = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 3 & 2 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\mathcal{A}(x) = (13, 21, 5)$$

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 13 \\ 21 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 13 \\ 2 & 1 & 3 & 2 & 21 \\ 0 & 1 & 1 & 0 & 5 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 13 \\ 0 & -1 & -1 & 0 & -5 \\ 0 & 1 & 1 & 0 & 5 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 13 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$x_4 = t$$

$$x_3 = s$$

$$t, s \in \mathbb{R}$$

$$x_2 = 5 - s$$

$$x_1 = 13 - s - t$$

$$x = \begin{pmatrix} 13 \\ 5 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Lineární zobrazení

$$A: L_1 \rightarrow L_2, B: L_2 \rightarrow L_3$$

$A \circ B: L_1 \rightarrow L_3$ lineární zobraz, které je definováno předpisem $(A \circ B)(x) = B(A(x))$, $\forall x \in L_1$

Pr Vypočítejte $AB(2,1)$, kde $A: \mathbb{R}^2 \rightarrow \mathbb{R}$; $B: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

$$\begin{aligned} A(2,1) &= 5 & B(3,-2) &= (4,13) \\ A(0,1) &= -1 & B(1,0) &= (2,3) \end{aligned}$$

$$\begin{aligned} e_1 &= (3, -2) \\ e_2 &= (1, 0) \end{aligned} \quad \left. \vphantom{\begin{aligned} e_1 \\ e_2 \end{aligned}} \right\} \text{ báze v } \mathbb{R}^2$$

$$x = x_1 e_1 + x_2 e_2$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = x_1 \begin{pmatrix} 3 \\ -2 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{aligned} x_1 &= -\frac{1}{2} \\ x_2 &= \frac{7}{2} \end{aligned}$$

$$B(2,1) = -\frac{1}{2} \begin{pmatrix} 4 \\ 13 \end{pmatrix} + \frac{7}{2} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 \\ 4 \end{pmatrix}}}$$

$$\begin{aligned} f_1 &= (2, 1) \\ f_2 &= (0, 1) \end{aligned} \quad \left. \vphantom{\begin{aligned} f_1 \\ f_2 \end{aligned}} \right\} \text{ báze v } \mathbb{R}^2$$

$$A(B(x)) = A(y) = y_1 A(f_1) + y_2 A(f_2)$$

$$\begin{pmatrix} 5 \\ 4 \end{pmatrix} = y_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + y_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \begin{aligned} y_1 &= \frac{5}{2} \\ y_2 &= \frac{3}{2} \end{aligned}$$

$$A(B(2,1)) = A(5,4) = \frac{5}{2} \cdot 5 + \frac{3}{2} \cdot (-1) = \frac{22}{2} = \underline{\underline{11}}$$

Pr $E = (e_1, e_2)$ je usporádk. báze V

$F = (f_1, f_2, f_3)$ — " — W

$$\alpha: V \rightarrow W \quad A = \begin{pmatrix} 3 & 2 \\ 1 & -1 \\ 2 & 5 \end{pmatrix}$$

Overle: $G = (e_1 - e_2, e_1 + e_2)$ je báze na V ?

$H = (f_1, f_1 + f_2, f_1 + f_2 + f_3)$ — " — W ?

$B = ?$ matice vzhledem k těmto bázím

$$\begin{aligned} e_1 = (1, 0) &\rightarrow g_1 = (1, -1) \\ e_2 = (0, 1) &\rightarrow g_2 = (1, 1) \end{aligned} \quad \left. \vphantom{\begin{aligned} e_1 = (1, 0) &\rightarrow g_1 = (1, -1) \\ e_2 = (0, 1) &\rightarrow g_2 = (1, 1) \end{aligned}} \right\} \begin{array}{l} \text{báze na } V \\ \text{stejný vzhledově} \end{array}$$

$$\forall x \in V \quad x = x_1 e_1 + x_2 e_2 = x'_1 g_1 + x'_2 g_2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}$$

$$\forall y \in W \quad y = y_1 f_1 + y_2 f_2 + y_3 f_3 = y'_1 h_1 + y'_2 h_2 + y'_3 h_3$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y'_1 \\ y'_2 \\ y'_3 \end{pmatrix}$$

$$A \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix}$$

$$B \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} = \begin{pmatrix} h'_1 \\ h'_2 \\ h'_3 \end{pmatrix}$$

$$A \cdot \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} y'_1 \\ y'_2 \\ y'_3 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \cdot A \cdot \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}}_B \cdot \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} = \begin{pmatrix} y'_1 \\ y'_2 \\ y'_3 \end{pmatrix}$$

B

$$B = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 2 \\ 1 & -1 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -1 & -6 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 5 & -7 \\ -3 & 7 \end{pmatrix}$$

Skalární součin

L ... lineární prostor, operace $L \otimes L \rightarrow \mathbb{R}$ je skalární součin

Velikost vektoru

Pro $x \in L$ def. jeho velikost

$$|x| = \sqrt{x \cdot x}; \quad |x|^2 = x \cdot x$$

Úhel 2 vektorů

$$x, y \in L, \quad x \neq 0, \quad y \neq 0$$

$$\cos \varphi = \frac{x \cdot y}{|x| \cdot |y|}$$

Kolmost vektorů

$$L \dots, \quad x, y \in L$$

Vektory x, y jsou na sebe kolmé ($x \perp y$)

$$\text{jestliže } x \cdot y = 0$$

Ortogonalní báze

$(B) = (b_1, b_2, \dots, b_m)$ na lin. prostoru L

Báze B je ortogonální, když $b_i \perp b_j \quad \forall i \in \{1, 2, \dots, m\}$
 $\forall j \in \{1, 2, \dots, m\}$
 $i \neq j$

Pozn. ortonormální báze $\wedge \forall$ je označ. jako $\{i, j, k\}$

$$i \cdot i = 1, \quad i \cdot j = 0 \dots i \cdot k = 0 \dots k \cdot k = 1$$

$$x = x_1 i + x_2 j + x_3 k \text{ má souřadnice}$$

$$x = (x_1, x_2, x_3) = (x \cdot i, x \cdot j, x \cdot k)$$

Př.

$$a = k - j - i$$

$$b = k - j$$

$$c = i - k$$

$$\angle(a, b) = ?$$

$$\angle(b, c) = ?$$

$$\angle(a, c) = ?$$

$$\cos \angle(a, b) = \frac{a \cdot b}{|a| \cdot |b|}$$

$$|a| = \sqrt{(j-i) \cdot (j-i)} = \sqrt{j \cdot j - 2 \cdot j \cdot i + i \cdot i} = \sqrt{2}$$

$$|b| = \dots \sqrt{2} = |c|$$

$$a \cdot b = \dots -1$$

$$b \cdot c = -1$$

$$a \cdot c = -1$$

$$\frac{2}{3}\pi + 2k\pi$$

$$\frac{4}{3}\pi + 2k\pi$$

$$a = -2i - 3j + k$$

$$b = -i + 2j - 3k$$

$$x = ? \quad x \perp a$$

$$x \perp b$$

$$\underline{x \cdot (-2i - j + k) = 2}$$

$$x \cdot (-2i - 3j + k) = 0$$

$$-2 \underbrace{x \cdot i}_{x_1} - 3 \underbrace{x \cdot j}_{x_2} + \underbrace{k \cdot x}_{x_3} = 0$$

$$(1) \quad -2x_1 - 3x_2 + x_3 = 0$$

$$(2) \quad -x_1 + 2x_2 - 3x_3 = 0$$

$$(3) \quad \underline{2x_1 - x_2 + x_3 = 2}$$

$$\begin{pmatrix} -2 & -3 & 1 & | & 0 \\ -1 & 2 & -3 & | & 0 \\ 2 & -1 & 1 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 3 & | & 0 \\ -2 & -3 & 1 & | & 0 \\ 0 & 3 & -5 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 3 & | & 0 \\ 0 & -7 & 7 & | & 0 \\ 0 & 3 & -5 & | & 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -2 & 0 & | & 6 \\ 0 & -1 & 1 & | & 0 \\ 0 & 0 & -7 & | & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & -1 \end{pmatrix} \quad ? \quad 1$$

$$\underline{x = i - j - k}$$

Vektorový součin

je roven $L \times L \rightarrow L$

a) $x, y \in L \quad L^2 \rightarrow x \times y = 0$ (nul. vektor)

b) $LN \rightarrow x \times y: 1) (x \times y) \perp x, (x \times y) \perp y$

2) $|x \times y| = |x| \cdot |y| \cdot \sin \varphi$

$u, v \in \mathbb{R}^3, u = (u_1, u_2, u_3)$

$v = (v_1, v_2, v_3)$

$$u \times v = \begin{pmatrix} |u_2 & u_3| & |u_1 & u_3| & |u_1 & u_2| \\ v_2 & v_3| & v_1 & v_3| & v_1 & v_2| \end{pmatrix}$$

$u, v \in L, \alpha \in \mathbb{R}$

1) $u \times u = 0$

2) $u \times v = -(v \times u)$

3) $(\alpha \cdot u) \times v = \alpha \cdot (u \times v)$

4) $(u \times v) \times w = u \times w + v \times w$

zjednodušíš

$$a \cdot ((a+b) \times (a+b+c)) =$$

$$= a \cdot \left(\underbrace{a \times a}_0 + a \times b + a \times c + \underbrace{b \times a}_{-(a \times b)} + \underbrace{b \times b}_0 + b \times c \right) =$$

$$= \underbrace{a \cdot (a \times c)}_0 + a \cdot (b \times c) = a \cdot (b \times c)$$

jezero na sebe
kolmé

geometrický význam vekt. součinu
 $u, v \in \mathbb{R}^n$

$$|u \times v| = S_{\square}$$



smíšený součin

$$a, b, c \in \mathbb{R}^3, \quad a \cdot (b \times c)$$

$$(B) = (i, j, k)$$

$$a = (a_1, a_2, a_3)_B$$

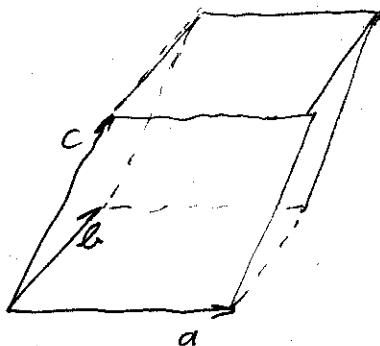
$\vdots$

$$a \cdot (b \times c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

geom. význam

$$|(a \times b) \cdot c| = V$$

$$= \underbrace{|a \times b|}_{\text{ráhla dna}} \cdot \underbrace{|c|}_{\text{výška}} \cdot \cos \varphi$$



$$\underline{\text{Pr}} \quad V_{ABCD} = ? \quad \vec{AB} = 3a - b$$

$$\vec{AC} = b + c$$

$$\vec{AD} = a - c$$

$$|a| = 2 \quad a \perp b$$

$$|b| = 1 \quad a \perp c$$

$$|c| = 3 \quad \angle(b, c) = \frac{\pi}{6}$$

$$V_{ABCD} = \frac{1}{6} V_{\text{normobēnristēna}}$$

$$(\vec{AB} \times \vec{AC}) \cdot \vec{AD} = ((3a - b) \times (b + c)) \cdot (a - c) =$$

$$= (3a \times b + 3a \times c - b \times b - b \times c) \cdot (a - c) =$$

$$= \underbrace{(3a \times b) \cdot a}_0 + \underbrace{(3a \times c) \cdot a}_0 - (b \times c) \cdot a - (3a \times b) \cdot c -$$

$$- \underbrace{(3a \times c) \cdot c}_0 - \underbrace{(b \times c) \cdot c}_0 =$$

$$= -(b \times c) \cdot a - (3a \times b) \cdot c = -(b \times c) \cdot a - 3(b \times c) \cdot a =$$

$$= -4(b \times c) \cdot a$$

$$V_{ABCD} = \frac{4}{6} (b \times c) \cdot a = \frac{4}{6} |b \times c| \cdot |a| \cdot \underbrace{\cos 0}_1 = \frac{4}{6} |b| \cdot |c| \cdot |a| \cdot \sin \angle(b, c) =$$

$$= \frac{4}{6} \cdot 2 \cdot 1 \cdot 3 \cdot \frac{1}{2} = \frac{12}{6} = \underline{\underline{2}}$$